User`s manual

2D Simulation of Dynamics of Granular Media

2020

Contents

14. 2D Simulation of dynamics of granular media

14.1. Introduction

Module **UM Ballast** is intended for research of dynamics of planar model of railway ballast and any other granular media. The module allows to research processes of forming of texture of railway bed under the work of the ground machines and solves tasks of optimization of fraction composition of ballast, kinematical parameters of tips of the railway machines and technology of their employment. The following tasks are solving with the help of **UM Ballast**:

- Filling the arbitrary volume (hard box or volume of difficult form set by a user);
- determination of eigenfrequencies of granular media;
- compaction of ballast under the action of vibrations of hard box;
- simulation of interaction between the ballast and other objects such as ties, blades of ballast-tampers etc.

In the next chapter short theoretical information on mathematical models and methods, used for research of dynamics of ballast is described. It includes the following topics:

- mathematical model of granular media;
- model of contact interaction;
- polygons collision detection algorithm;
- features of integration of stiff equations of motion of the system with the help of approximate Jacobians.

The third chapter describes creating model in the software package. The typical workflow is as follows (*UM Input*):

- creating a subsystem of Ballast type;
- creating a fictitious body;
- features of creation of objects, interacting with the ballast media;
  and in the processor/postprocessor (*UM Simulation*):
- setting hard box parameters;
- setting granular media parameters;
- creating granular media particles;
- setting vibrocompaction parameters.

The recommended tuning of numerical method, simulation modes and types of output information, available during integration of equations of motion of the system is presented in the fourth chapter.

A final, fifth chapter describes creation of the simplest examples that illustrates how to work in **UM Ballast**.

14.2. Theory

Railway ballast is a granular media. There are several principally different approaches to the simulation of granular media. Basic of them are as follows:

- the simulation of granular material as continuous medium (for example, with the use of finite element method (FEM));
- the solid-state model (discrete model, which consists of the particle-granules, the method of discrete elements (DEM)).

The first approach is limited in application because it does not take into account geometry of particles of the ballast. However the second approach as a rule requires considerably more CPU expenses in the usual usage. The second, DEM-based approach is used in the **UM Ballast** module, so ballast is considered as a discrete granular environment.

Real system   Discrete model

Continuous model   Unilateral   Compliant
                            constraints   forces
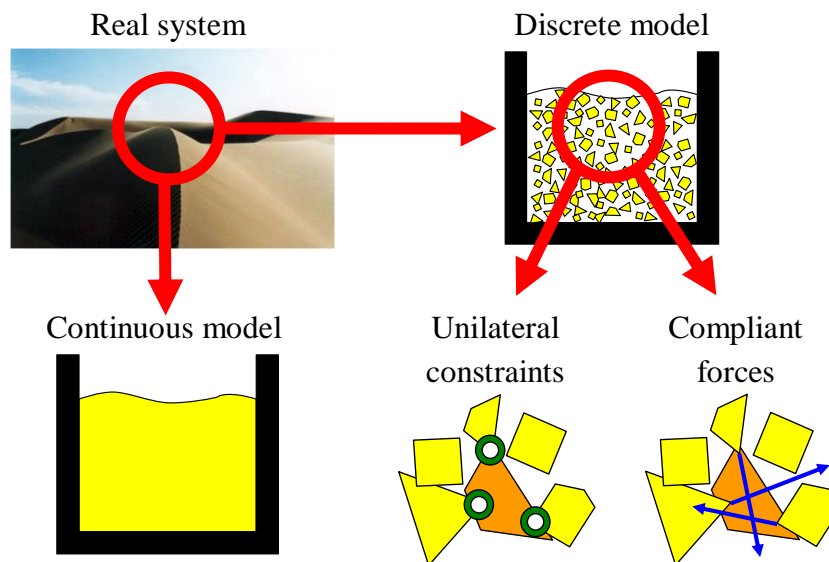
The particles of a granular media are usually considered as rigid bodies. The first approximation of such a model is the system of hard spheres in 3D case or circles in 2D one. The great number of such models is described in literature, that explained by simplicity of their realization. However an application of similar models is very limited and, mainly, spreads to the liquids and powders dynamics simulation. So, at the simulation of railway ballast, it is necessary, that bodies were presented by the acute-angled particles. It is explained by that behavior of model of ballast under vertical load should not be like behavior of liquid. The next approximation of planar granular media is the system of polygons. The calculations of polygons contact is an order higher on complication and time of implementation from an analogical calculation for spheres. Particles of granular media in the **UM Ballast** are polygons.

The basic elements of mathematical model of granular media are the following:

- model of the system of particles;
- model of contact forces;
- collision detection algorithm;
- features of integration of equations of motion.

Let us consider the description of mathematical model of a granular media. All bodies also called particles and granules, included in the system, are presented by flat polygons. A contact between bodies arises up, as particles' borders become overlapped. It results in appearance of contact interaction forces. In the planar model every body of the system has three degrees of freedom. Position of i-th granule is definitely determined by centre-of-mass coordinates x_{ci}, y_{ci} and rotation angle ϕ_i. Thus, a ballast media, consisting of N of granules, has 3N degrees of freedom, and its equations of motion are the following:

$$m_i \ddot{x}_{ci} = \sum_j F_{xij}^c,$$

$$m_i \ddot{y}_{ci} = -m_i g + \sum_j F_{yij}^c,$$

$$J_{zi} \ddot{\varphi}_i = \sum_j m_z\left(F_{ij}^c\right).$$

Here m_i, J_{zi} are a mass and a moment of inertia of i-th particle; $\mathbf{F}_{ij}^c$ is contact force, acting between i-th and by j-th granules. For simplicity of description here we consider, that the pair of granules contacts only in one point. A mass matrix of such a system is diagonal. It will be shown below, that usage of approximate block-diagonal Jacobi matrix accelerates a process of integration with the insignificant losses of accuracy.
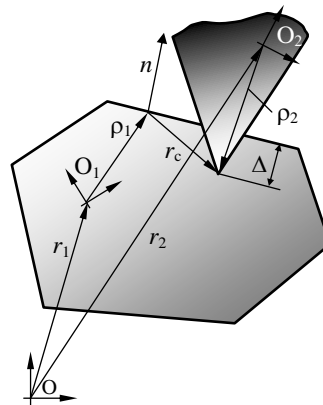

Figure 14.1. Determination of penetration and its velocity

Consider the mathematical model of contact forces interacting between bodies. The base of the model is this fact: contact forces appear in a vertex of a polygon if only it has penetration into another polygon, limiting a nearby granule. The force is a sum of normal and tangential parts. The normal force depends on the penetration Δ and its velocity $\dot{\Delta}$. The penetration Δ is calculated as a minimum distance from the embedded vertex to edges of another polygon. If the model has polygons with acute angles, its simulation gives incorrect result, therefore acute angles are cut away in the vicinities of corresponding vertices. As such contact force depends on distance between a vertex and one of edges of polygon, we will call this interaction as contact of point-line type. In fact, this interaction is a special case of spatial contact interaction of point-plane type (see Chapt. 2, Sect. 6.7.1), that found a wide use at the simulation of dynamics of freight cars. The force consists of two parts: normal reaction N perpendicular to the contact line, and friction force F, lying on this line. As well as in case of contact of point-plane type, the linear viscous-

elastic model of normal force $N = -c\Delta - \mu\dot{\Delta}$ with the constant coefficients of contact stiffness c and damping μ is used. The Figure 14.1 shows that the penetration can be determined as follow:

$$\Delta = n^T r_c, \quad r_c = r_2 + A_{02}\rho_2^2 - r_1 - A_{01}\rho_1^1.$$

The penetration velocity is calculated by the following formula:

$$\dot{\Delta} = n^T\left(v_2 + \tilde{\omega}_2\rho_2 - v_1 - \tilde{\omega}_1(\rho_1 + r_c)\right) = n^T v_{c12},$$

where: n – is a normal to the contact line; r_1, r_2 are radii vectors of centre-of-mass of the first body (containing a contact point) and the second body (contact line); A_{01}, A_{02} are matrices of orientation; p_1, p_2 are the radius vectors of the contact point and a point of contact line in the local coordinate systems of corresponding bodies; v_{c12} is a vector of relative velocity of the point of the second body.

The model of force of friction has two modes: sliding and sticking. In the sliding mode this force is directed against the projection of relative velocity of sliding on a contact line:

$$F_f = -fN(e_s^T v_{c12})e_s,$$

where f is a coefficient of sliding friction; e_s is a unit vector, directed on a contact line. The sticking mode and conditions of mode switching is described in the Chapt. 2, Sect. 6.7.1. Coefficients of stiffness c and damping μ in the expression for normal force N can be either directly set or determined by the given partial contact frequency k_c and the damping ration β. Expressions, relating all these parameters, coming from the solving of task for the simplified contact of two nearby granules (Figure 14.2)

$$c = \frac{m_1 m_2}{m_1 + m_2} k_c^2; \quad \mu = 2\beta\sqrt{c\frac{m_1 m_2}{m_1 + m_2}}.$$

The value of partial frequency is chosen so that the eigenfrequencies of ballast media are higher than frequency of vibrocompaction. Now let us consider the collision detection algorithm.
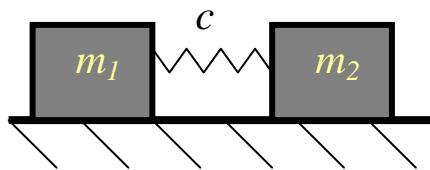

Figure 14.2. Task for determination of contact stiffness and damping on given contact frequency and damping ratio.

In the existent collision (crossing, overlapping) detection algorithms almost always a task is divided on two (or more) separate steps: collision detection of rough shells circumscribing particles, (so-called "far contact test") and collision detection of real geometrical shapes of particles ("near contact test"). These two tasks are also decided in several ways, depending on an application domain. For example, it is possible to choose a rough shell as a circle for the far contact of polygons. In this case a fact of crossing of two shells is determinated by this condition: $\|r_i - r_j\| \le R_i + R_j$, where R_i, R_j, r_i, r_j are radii and radius-vectors of centers of the shells. The main lack of this method is a need of verification of this condition on every step for every pair of the

bodies, therefore complexity of this method has quadratic order on the particle count. Method of the Linked Linear Lists [1] is considerably more rapid and has linear order on a body count. The idea of the method is the following. A rectangle of minimal area bounding a body with sides parallel to axes is a rough shell in this case.
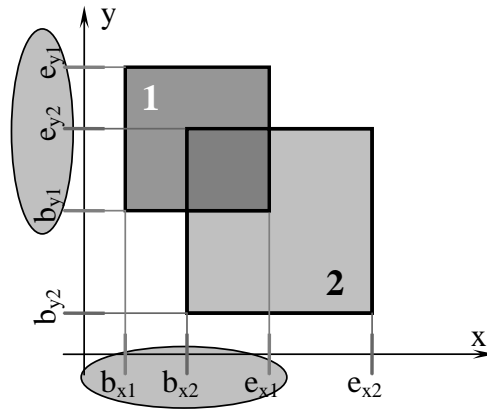

Figure 14.3. Illustration to the method of the constrained linear lists.

Such shell is determined by four parameters (in the planar case): begin b_{xi} and end e_{xi} of the shell projection on the x-axis, begin b_{yi} and end e_{yi} on the y-axis. Then for every axis an ordered list of these projections is calculated. Shells of two bodies are intersected if even one of projection of the first body lies between two ones of the second body for every axis. If so then a "near contact" is possible. These lists are sorted on every step of integration. If two some borders in a list were swapped around, then the following events are possible: 1) begin of one shell and begin of another one are swapped, 2) end of one shell and begin of another one are swapped, 3) swapping of two ends and 4) begin of one shell and an end of another. First and third events are not considered because of their useless. The second case means possibility of appearance of new contact. This appearance checked up on a list for other axis. If a contact is confirmed, the pair of bodies is checked for a near contact. A fourth event talks that if these bodies were contact on a previous step, then on this step they went out from a contact and there is no reason to check this pair for a near contact now.

For determination of near contact, i.e. collision (overlapping, intersection) detection of polygons, there are a few methods also. We will consider two rapid algorithms. First one is an Lin-Canny algorithm [2], based on the use of the so-called Voronoy-regions. This method effectively allows determining distance between polygons. Unfortunately, application of this algorithm is effective only in case of convex polygons. The second lack (for described contact interaction model) is that it cannot be used for collision detection of overlapping of polygons. Therefore it perfectly befits for the models of contact interaction in impact-based methods for small number of particles. Another method is based on determination of the fact of including a point into a polygon. Here number of crossing points of ray, passing through the point, with polygon edges is counting. If this number is odd then there is an intersection. The description can be found in [1]. The contact interaction model used in the described module **UM Ballast** requires determination of penetration Δ and its velocity. The approach above cannot give such a data. Therefore an alternative rapid algorithm is applied in the module.

Essence of the algorithm consists of the following: two polygons are crossed, if even one vertex of first one lies in sensitivity area of the second one, or vice versa. The pair of polygons on a Figure 14.4 will be considered as non-crossed, because all the vertices are in outside.
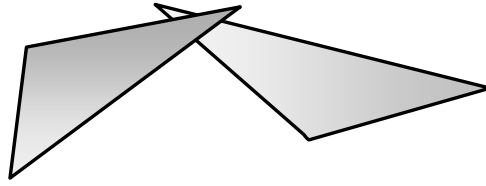
Figure 14.4. Special case of crossing of polygons.

Please note the following topics.
1. In the tasks of simulation of railway ballast dynamics, stuff of particles has relatively large modulus of elasticity. So, the values of penetration of particles will be small.
2. In addition, it was higher described, that acute angles are cut away.

It allows using this algorithm for simulation of granular media dynamics. We call the offered algorithm as the method of sensitivity area (SA). Now let us consider what is a SA. Let numeration of polygon vertices has counterclockwise order. Before the beginning of simulation for every polygon the array of data of the length equal to the number of vertices n_v is created. Every array element with contains (Figure 14.5) radius-vector of a vertex in local coordinate system of body-particle ρ_t, unit vector, directed along an edge of polygon e_i to the next vertex $v_{i+1}(v_{n_v+1} \equiv v_1)$, vector of outward bisector of adjoining corner b_i, depth of a sensitivity area of the nearby edge δ (this empiric constant is some little part of character size of the polygon), length of the edge l_i. The sensitivity area is an area limited by (1) the edge, (2) a line parallel the edge and lying inside a polygon in the distance of sensitivity depth, and (3) the pair of bisectrices of nearby corners.
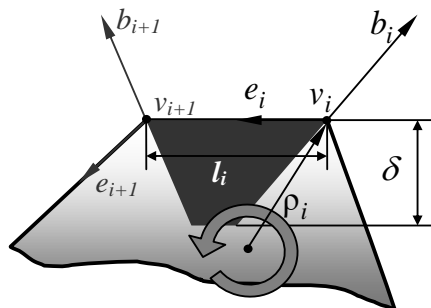
Figure 14.5. Sensitivity area of vi edge and its parameters.

The point lies in a sensitivity area if all oriented distances[1] between it and bounds of SA are positive.

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[1] Here oriented distance is a distance from a point to a line signed "+" if point lies in the left half plain (relatively to the line) and "−" otherwise.

When a railway ballast prism is being formed lower layers of a granular media are interacting with ground base. There is a so called "top layer of ground" model to take into account these contact forces.
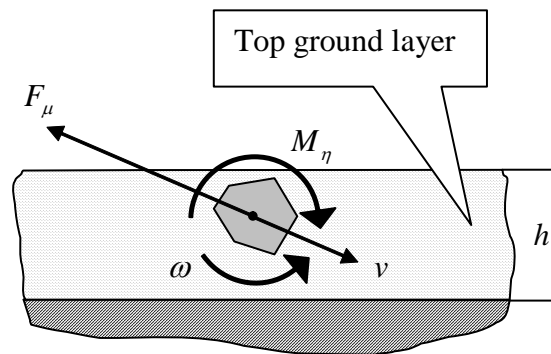
The mathematical model of the top layer is a simple viscous resistance force. The model has one geometrical parameter – height and two physical parameters –coefficients of linear and angular damping. If a particle comes into the top layer (particle center of mass is lower than the height of the layer) resistance force and moment begin acting on it:

$$\mathbf{F}_\mu = -\mu \mathbf{v};$$
$$M_\eta = -\eta \omega;$$

where μ is a coefficient of viscous friction, $\mathbf{v}$ is a vector of speed of the center of mass examined particle of ballast, η is a coefficient of viscous angular friction, ω is an angular velocity of the particle. So, on a particle moving on foundation acting: gravity, normal contact forces, dry contact friction force and force and moment of viscous resistance.

Equations of motion of such systems as railway ballast are stiff. Therefore one has to use implicit integration numeral methods for simulation. These methods require calculation of Jacobi matrices of right parts (forces) of equations of motion of the system (in this work Jacobi matrices of contact force elements are meant). The system matrix of a granular system has very wide profile, too wide for effective work of integration methods. Using approximate block-diagonal Jacobi matrices for integration of equations of system motion is considerably increases computational capability of integration methods and simultaneously saves sufficient accuracy [3] [4] [5].

14.3. Creating models

A model consists of the granular media and a container for it called a hard box. These objects are included in a subsystem of **Ballast** type. There is ability of interaction between bodies of main subsystem and ones of this subsystem. But any interactions with elements of other subsystems are not supported.

Subsystem **Ballast** is a dynamic subsystem. It means that bodies-particles and joints for them are created in the simulation program (processor/postprocessor **UM Simulation**). Although a subsystem is added to the model in the data input program (preprocessor **UM Input**), then, after entering of parameters of hard box, granular media and so on, the body-particles are dynamically

created in the simulation program *(UM Simulation)*. This feature allows generation the different variants of granular media, directly from the simulation program.
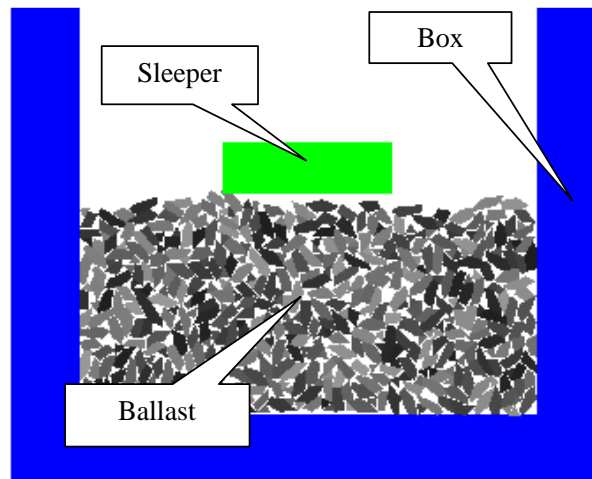

The workflow of creation of ballast model is as follows:

1. Data input program *(UM Input)*:
    *   creating of a new model in the data input program;
    *   creating of a subsystem of **Ballast** type;
    *   creating of a fictive body or bodies, interacting with a ballast (and corresponding joints and so on);
2. Simulation program *(UM Simulation)*:
*   setting the parameters of a hard box;
*   setting the parameters of granular media;
*   generation of elements of the **Ballast** subsystem.
    The detailed description is given below.

14.3.1. Data input program (UM Input)

Run the data input program *UM Input*. Click on the **Subsystems** of the list of elements and add a new subsystem (button ![btn]). Choose **Ballast** type of subsystem.
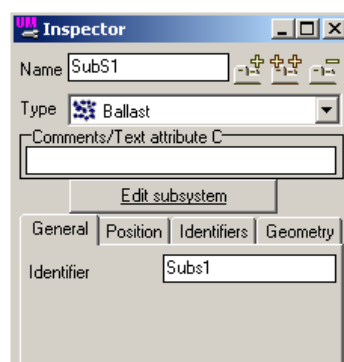

The generation of equations of motion is possible if only even one body there is in the model. The subsystem of **SubS1** is empty yet. Therefore it needs to add a fictitious body (not always, see below). User can add it by clicking ![btn] button on the inspector panel after selecting **Bodies** in

the list of elements. Rename it as **FictiveBody** and link by a joint of generalized type with the base. It is enough, that this joint has one elementary transformation of **TC** type.
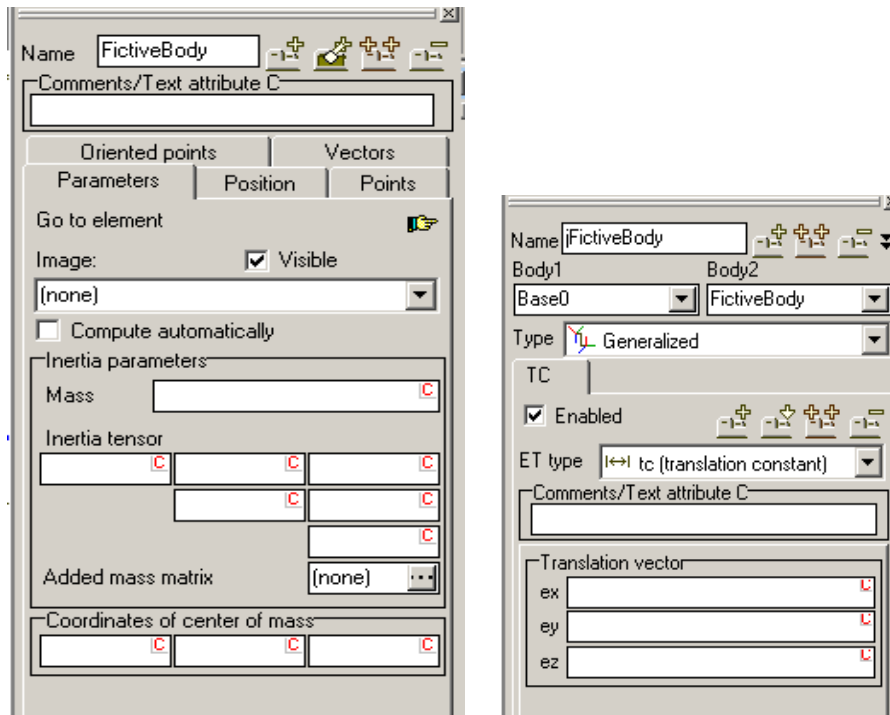
Fictive body is not necessary, if there are bodies interacting with the ballast, such as railroad ties, working tips and so on. Please note that these bodies included into the basic system, but not in any subsystem. Let us consider the features of creation of graphic object of these bodies.

A mathematical model of contact interaction of particles with surrounding objects is a model of contact of polygons. Therefore graphic objects, interacting with a granular media, should have the first graphical element of **Polyhedron** type. Moreover the order of numbering of vertices of the polyhedron should be counterclockwise. The example of the correctly described polyhedron included in the graphic object **WorkTool** is presented on a figure below.
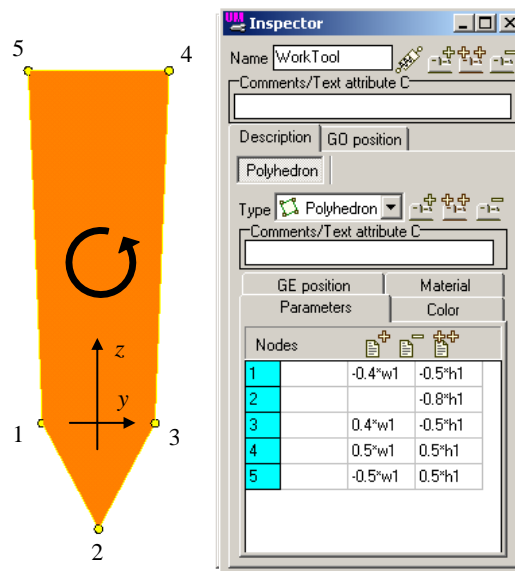
14.3.2. Simulation program (UM Simulation)

All parameters and settings of ballast and hard box are available on a dialog **Ballast parameters** which appears when user click button 🔳 of **Ballast** toolbar. Visibility of the toolbar is controlled by menu item **Instruments | Toolbars** or by the context menu of the toolbar. A dialog consists of a toolbar and five tab sheets. The toolbar contains the followings buttons:

- 🔄 performs dynamic generation of the granular media. Use it after input all parameters. Action of this button are: creating of particle bodies, their joints, setting initial particle positions, creating of walls and bottom of hard box and their joints;
- ▶ calls **Object simulation inspector**. Usually used after using of previous buttons;
- 📂 loads ballast parameters from a *.blt file;
- 💾 saves ballast parameters to a last *.blt file;
- 💾 saves ballast parameters to a new *.blt file;
- 🔳 opens **Ballast state** dialog. It is used for access to additional parameters of ballast state during model simulation (see Sect. 14.4.2.2. *"Simulation results for granular media"*, p. 14-20);
- 🔳 adds particles;

Parameters of subsystem **Ballast** grouped on the following tabs of the dialog:
- **Contact constants;**
- **Box;**
- **Infill;**
- **Compaction;**
- **Other.**

These parameters are usually setting in such order:
1. parameters of hard box;
2. geometrical parameters of granules;
3. parameters of initial position of granular media;
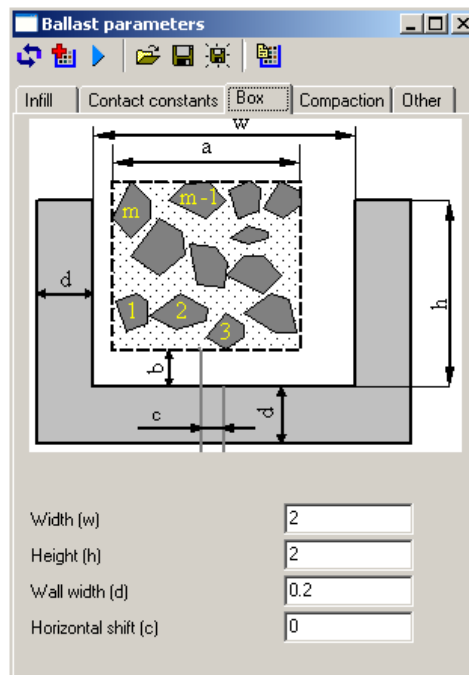4. parameters of vibrocompaction (if necessary).

Figure 14.6. **Ballast parameters** dialog.

14.3.2.1. Settings of parameters of a hard box

A box for granular media (hard box, boxing) consists of three bodies: left and right walls, and a bottom. The parameters of a box are grouped in geometrical ones (Figure 14.6) and parameters of the top ground layer.

There are following geometrical parameters:

- width of the filled area (m);
- height of the filled area (m);
- thickness of walls (m);
- horizontal shift of the box (m).

By using the last parameter **Horizontal shift (c)** and parameters of geometry of the granular media it is possible to get different configurations of initial position of ballast relative to the box.
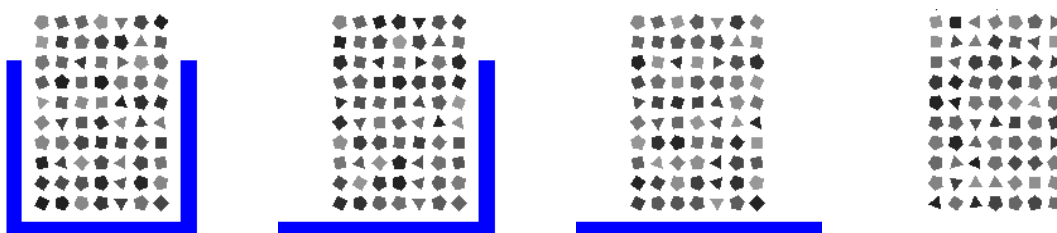
The mathematical model of the top ground layer is described in Sect. 14.2. *"Theory"*, p. 14-4. In the parameters of the ground layer included:

- coefficient of damping of a top ground layer, kg/s;
- coefficient of angular damping of the layer, kg*m/s;
- height of the layer, m.

Edit boxes for these parameters are on **Contact constants** tab of the **Ballast parameters** dialog.

| Coefficient of dissipation of damping layer | 7.5 |
| Coefficient of ang. dissipation of the layer | 1.3 |
| Height of damping layer | 0.04 |

All above-described parameters cannot be set by identifiers.

14.3.2.2. Setting of granular media parameters

A discrete model of friable (granular) media is a set of rigid particals. These particles interact with nearby ones and with other bodies by contact forces (see Chapt. 2, Sect. Contact forces). The parameters of granular media can be divided into the following groups:

- geometrical parameters of particle;
- geometrical parameters of media;
- parameters of feeding of particles;
- physical parameters of interaction of particles among themselves;

All these are set on the tabs of **Ballast parameters** dialog.
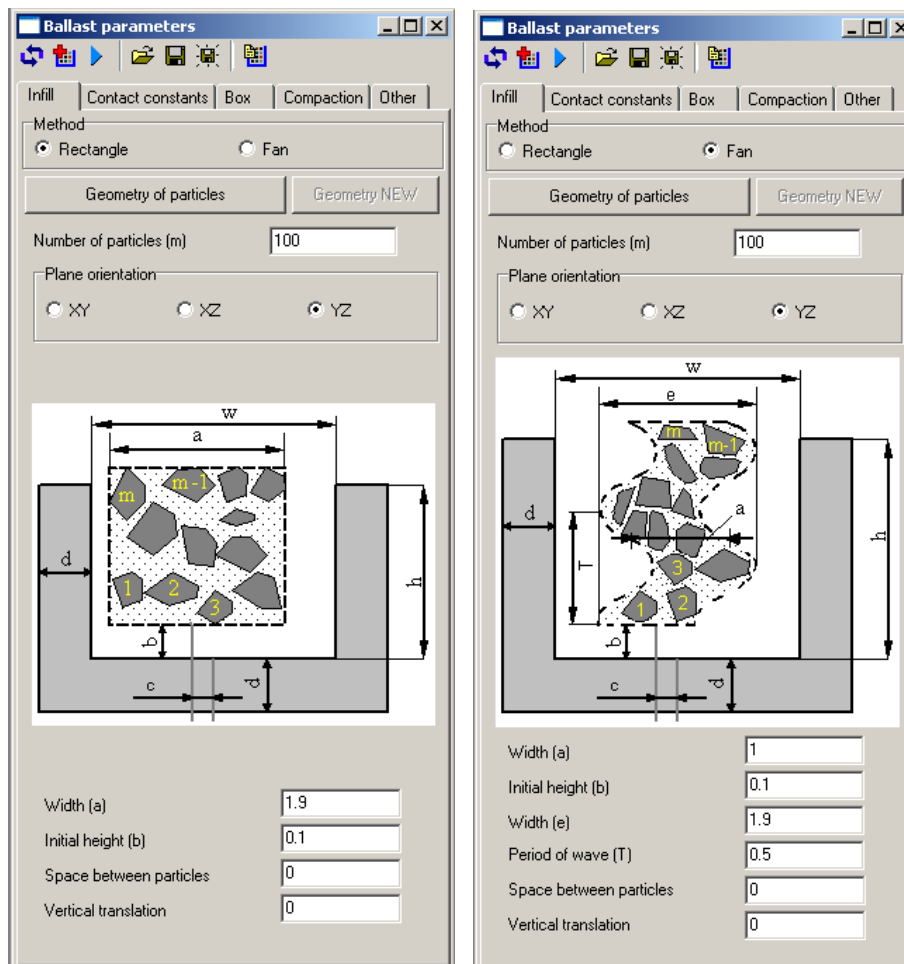
a)   b)

Figure 14.7. Infill tab

As it is written above the model of granular media is a planar one. Therefore user needs to point the plane of the **Ballast** subsystem. **YZ**-plane is set for it by default. In the case of simulation of longitudinal dynamics of a railway freight car body interaction with a friable soil inside it (for example during collision of vehicles), the user should us **XZ** plane.

14.3.2.2.1. Geometry of particles

Geometry of particles dialog allows to set statistics of geometrical parameters of particles of granular media. The dialog arrears by clicking button of the same name, which is placed on **Infill** tab of **Ballast parameters** dialog.

- Setting of a number of different types of polygons (**Number of types of polygon**). A type in this case means a number of vertices. For example, if a granular media consists of only triangles, then this number is equal to 1. If it includes quadrangles too, then the number is 2.
- Setting of histogram of distribution on the number of vertices. The histogram has a view of a table. A number of types of polygons (see above) is a number of this table columns. The first row of the table is a number of vertices, the second one is a rate of a corresponding fraction, and the last row is a density of particles of the fraction. The case on the picture below represent situation, when a granular environment consists of fifth part of triangles, three fifth of quadrangles and the rest fifth of pentagons.
- Setting of histogram of characteristic sizes. This histogram shows distribution of particles with the corresponding number of vertices on characteristic sizes. Rows of it correspond to the vertical characteristic size, columns correspond to horizontal one. Values of probabilities in a cell are a rate of particles with corresponding characteristic sizes in the corresponding fraction. A table **Elements**, edit box **Element count** and drop-down **Characteristic dimension** allow to edit combinations of characteristic sizes. **Vertices count** text label specifies on polygon fraction, edited distribution of characteristic sizes.
- Setting of convexity. Check box **Convexity** is responsible for possibility of generation of convex/nonconvex particle with the corresponding vertices number.
- Tuning irregularity of particles form. This parameter determines how the form of particle will be near to correct (0 is a regular shape, 1 – near to correct, et cetera).
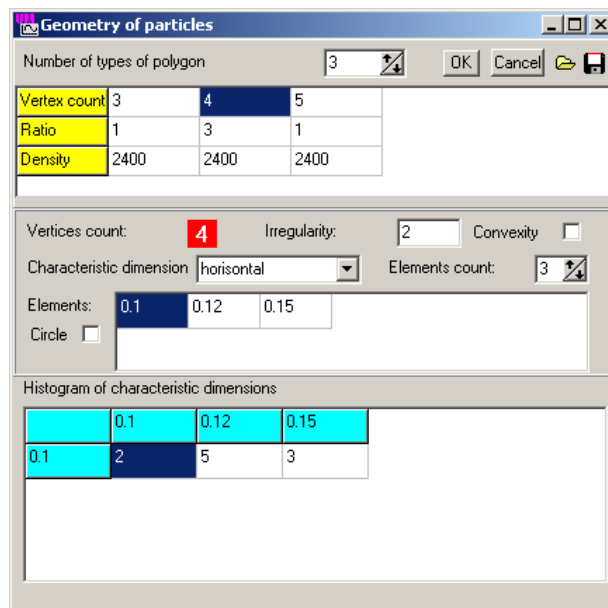
Figure 14.8. A dialog is Geometry of particles

User can save parameters of statistics of geometry of particles in a file using the button 🖫. File of statistics of geometry has expansion of **bst**. Using the button 📂 it is possible to load the files of statistics created before. After the setting of parameters of statistics of geometry user can accept (button **OK**) or decline (button **Cancel** or key **Escape**) changes. A picture explaining the value of parameters of dialog **Geometry of particles** on a Figure 14.8 is presented below.
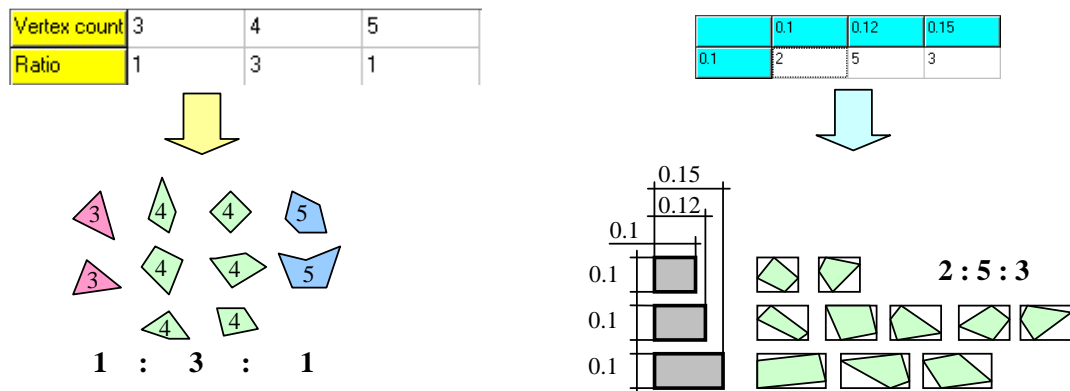

14.3.2.2.2. Geometry of granular media

Initial position and form of granular environment are set on **Infilling** tab. An initial form of granular environment can be rectangular or sinusoidal. It is controlled by **Method** radio group (see a Figure 14.7). A rectangular form is set by two parameters: a width and an initial height of media bottom (distance from the bottom of box to the first layer of particles of ballast). Sinusoidal form besides a width and an initial height is also set by filling width and wavelength. All parameters are presented on the explaining pictures of **Infilling** tab, see a Figure 14.7.

UM uses these parameters and parameters of statistics of geometry of particles for the initial placing of particles, avoiding overlapping of particles between each other.

14.3.2.2.3. Setting of media feeding

The preliminary stage of simulation of a granular media is filling of some initial volume. In some tasks there are so many particles, that some of them are set very high. Particles of media are under gravity during the filling stage. Therefore top layers of falling particles achieve very high speed which causes of many impacts at landing moment. However this situation is not good for numerical integration of motion of the system. User shall set parameters of feeding of particles located on the **Contact constants** tab (see a Figure 14.9) to avoid such a problem. Gravity starts acting on a particle, if z-coordinate of its center of mass less than **Height of dropping of particles**. The figure explaining feeding is shown below.

Initial height

Height of dropping of particles
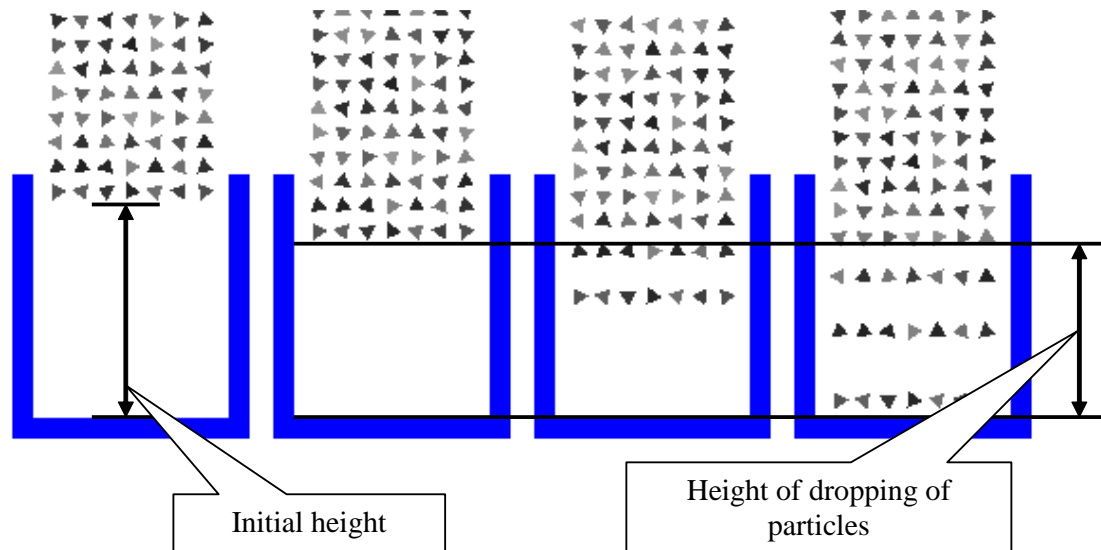
14.3.2.2.4. Setting of contact force parameters

Contact forces appear when shapes of two particles become overlapped. Every particle has a polygonal form, so overlap exists then a vertex of one polygon lays inside an edge of another one or vice versa (see Sect. 14.2. *"Theory"*, p. 14-4). Normal contact force lineary depends on penetration and its first time derivation (penetration velocity). The coefficients of elasticity, damping and coefficients of friction, are set on the **Contact constants** tab, see a Figure 14.9. There coefficients are a parameters for not one but a couple of interacting bodies. There is ability to set different coefficients for such pairs: particle-vertical wall, particle-horizontal wall, particle-sleeper (or any external object) and of course particle-particle. In addition for particle-particle contact type exists two methods of setting of physical parameters: **Automatically** and **Manually**. Manual method allows to enter all the coefficients manually in corresponding edit boxes, and automatical method calculates them on base of entered contact frequency and damping ratio of critical one, see a Figure 14.2.
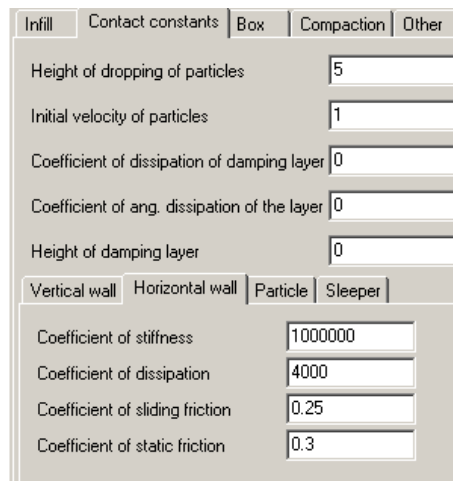
Figure 14.9. Physical parameters of contact

14.3.2.2.5. Correcting of initial positions of group of particles

Often situations appear during simulation when there is a need to correct initial positions of some group of particles (for example in a case some of particles moved off from the box). User shall use **Shift of selected particles** tool on **Other** tab for such a correction.
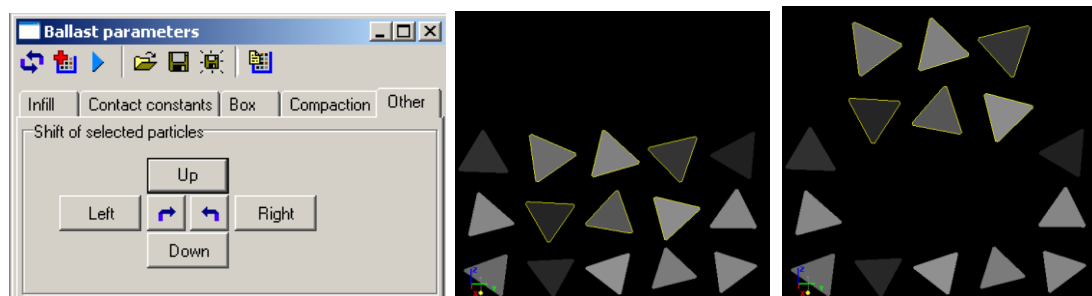

The only selected particles moved by clicking of buttons of this tool. Selection and deselection of a particle occurs when a user make left click with **Shift** key held. The example of action of **Up** button on six selected particles is shown in the figure above.

14.3.2.3. Parameters of vibrocompaction in a hard box

In some tasks main simulation is carried out on a compacted granular media (for example, packing under sleeper with blades of railroad machine). To achieve such a media the following parameters of compression are used:

- amplitudes of vibrations, m;
- frequency of vibrations, Hz.

These parameters are located on **Compaction** tab:
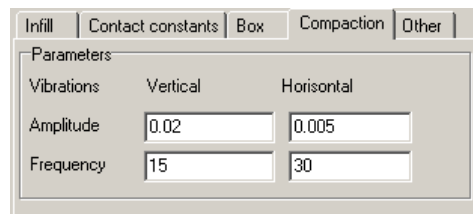
The law of motion of box looks like $A \cdot \sin(\omega t)$. By setting frequency ω or amplitude A to zero of one of vibrations it is possible to get the different variants of vibrocompaction: horizontal (zero frequency and/or amplitude of vertical vibrations), vertical (zero frequency and/or amplitude of horizontal vibrations), combined. If a vibrocompaction does not need to be used, all frequencies and amplitudes should zero (as default).

14.4. Simulation

Research of dynamics of ballast usually consists of the following stages.
- Simulation of filling process of the ballast in a hard box (or other volume, set by a user).
- Analysis of ballast media in a frequency domain. It gives the user eigenfrequencies of the media. To avoid resonance processes at a vibrocompaction stage it is necessary that contact frequencies are higher than eigenones. If this verification is going well, it is possible to pass to the next stage.
- Simulation of vibrocompaction in a hard box.
- Simulation dynamics of action of external objects on a ballast media (for example, experiments on determination of vertical and horizontal stiffness, where an external object is a railroad sleeper).

14.4.1. Simulation in frequency domain

A user shall use **Linear analysis** dialog to investigate model in frequency domain. It appears by clicking button ![lambda] on the integrator toolbar. The calculation of eigenfrequencies occurs after switching on **Frequencies** tab. Eigenfrequencies are sorted in ascending order. A linear analysis shall be made for models where a granular media is in equilibrium, or in other words after the filling stage.

14.4.2. Simulation in a time domain

Numeric integration of equation of motion of the system or simulation of the system dynamics includes three stages:
- filling the ballast in hard box or other volume, set by a user;
- simulation of media vibrocompaction by the walls of hard box;
- simulation of interaction of the granular media with different external objects, for example, tamping of ballast by shoulder-blades of ballast-tamper machine;

The workflow order cannot be changed because final position of the media in the first stage is initial position for the simulation of the second stage, et cetera.

Before simulation in any mode it is necessary to tune the parameters of numerical methods. It is also necessary to specify a name of XVA-file for the integration data recording if XVA mode is turned on. These files are very useful for animation clip creation and playing of simulation process in real-time (and faster), more details see in Sect. 14.4.2.3. *"XVA"*, p. 14-23.

14.4.2.1. Simulation process parameters

Equations of motion of granular media are stiff (because of contact forces). Therefore simulation of dynamics of ballast is impossible without using of implicit integration methods with calculation of Jacobi matrices. The recommended parameters of integrator are presented on a figure below.
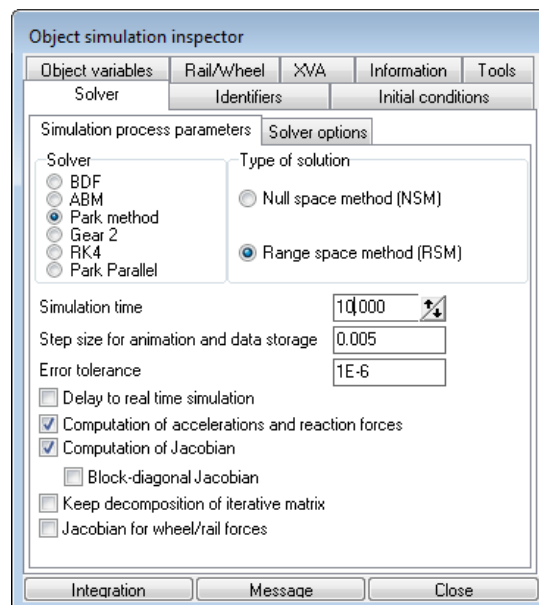

| Note.	In tasks with large number of particles integration of equation of motion of the system is impossible without the calculation of approximate block-diagonal Jacobian.

14.4.2.2. Simulation results for granular media

In addition to basic output data for mechanical system such as diagrams of particles motion kinematical parameter and so on, there is a possibility to analyze special characteristics for granular media. They are accessible in a **Ballast state** dialog. The dialog appears by clicking 🔳 button of the toolbar of the **Ballast parameters** dialog and includes the following tabs:

- **Kinematics.** On this tab the kinematics parameters of granular media are presented. This information can be used for determination of moment when filling of the media in hard box is finished.
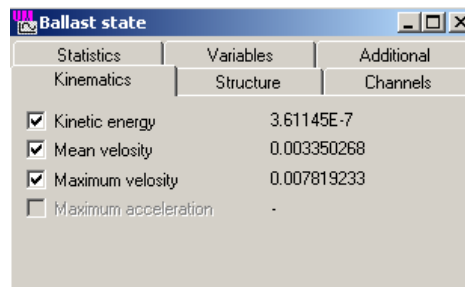
- **Statistics.** The different integral parameters of granular media are presented here.
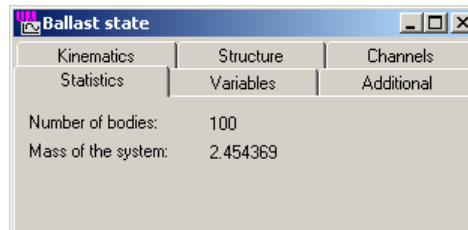
- **Structure.** Here are collected parameters for estimation of compaction of granular media (global and local porosity) and parameters, characterizing quality of vibrocompaction: shrinkage of ballast, top middle level of ballast. Some of these parameters are graphically illustrated in an animation window.
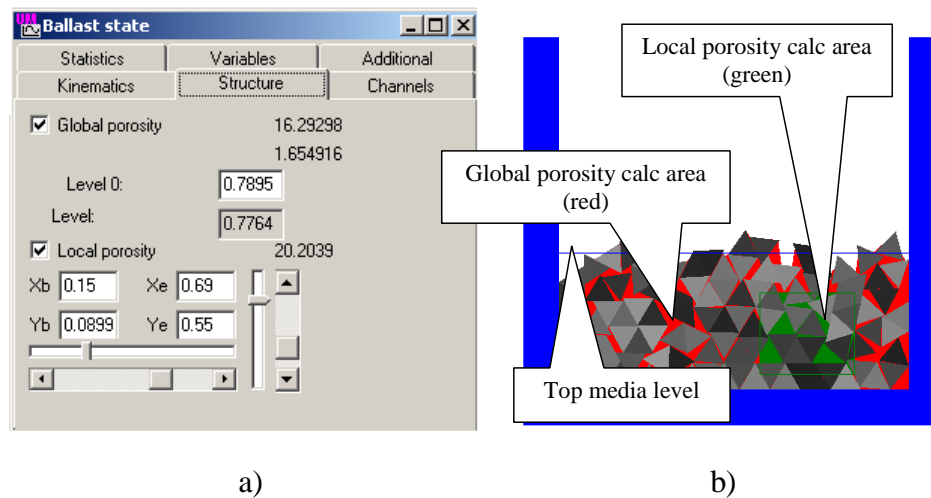

a)   b)

Figure 14.10. Information about structure of a ballast: a) **Structure** tab; b) graphic presentation of some parameters.

Porosity is a ratio of free space in percents. In flat case porosity is equal to:

$$\zeta = \frac{S_0}{S} = \frac{S - S_1}{S},$$

where S_0 is a total empty area, S_1 is a total area of particles, S is an area, occupied by a granular media $S = S_0 + S_1$. Rectangular area of granular media is considered to calculate local porosity. Global porosity is determined for all granular media area. The calculation of porosities is managed by corresponding check boxes. In addition, for local porosity calculation it is necessary to

specify a rectangular area. A user can make it by both the movable indicators of positions and sizes of an area, and by hand through the **Xb, Xe, Yb, Ye** edit boxes. Occupied by a granular media area is marked a red filling in an animation window. Working area for local porosity calculation is marked by green color there (S for local porosity). The **Shrinkage** parameter has a dimension of percents. It shows decreasing of top height of the medium during a simulation (level of ballast). Shrinking is calculated as:

$$\xi = \frac{H_0 - H}{H_0}$$

where H_0 is an initial level of ballast, H is a current level of ballast.

Note.	Values of the described here parameters do not make sense in ballast filling mode.

- **Colors**
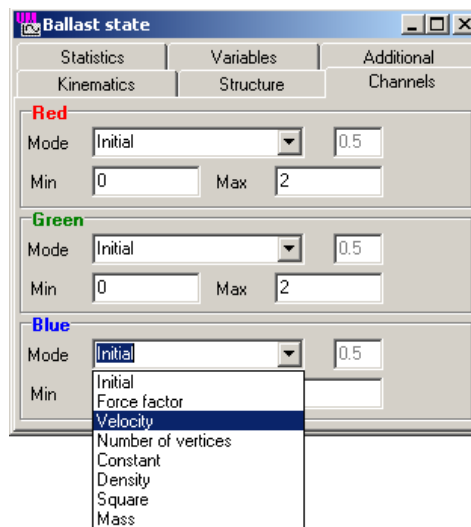
Figure 14.11. **Color** tab.

Besides the usual numerical output data, the module allows to obtain an image of distribution on such parameters of granular media as: force factor for particle (load), velocity of center of mass of a particle, count of vertices, and so on. All three color channels (RGB) are independent between each other. It means that, for example, if a user tune the red channel on a particle velocity and the green one on a force factor, then unloaded particles with high speed is painted in a red color, hard loaded slow or immovable particles – in green, and hard loaded rapid particles – in yellow (mix of red and green colors). As presented on a picture above every color channel also can be set by a constant or initial random number.
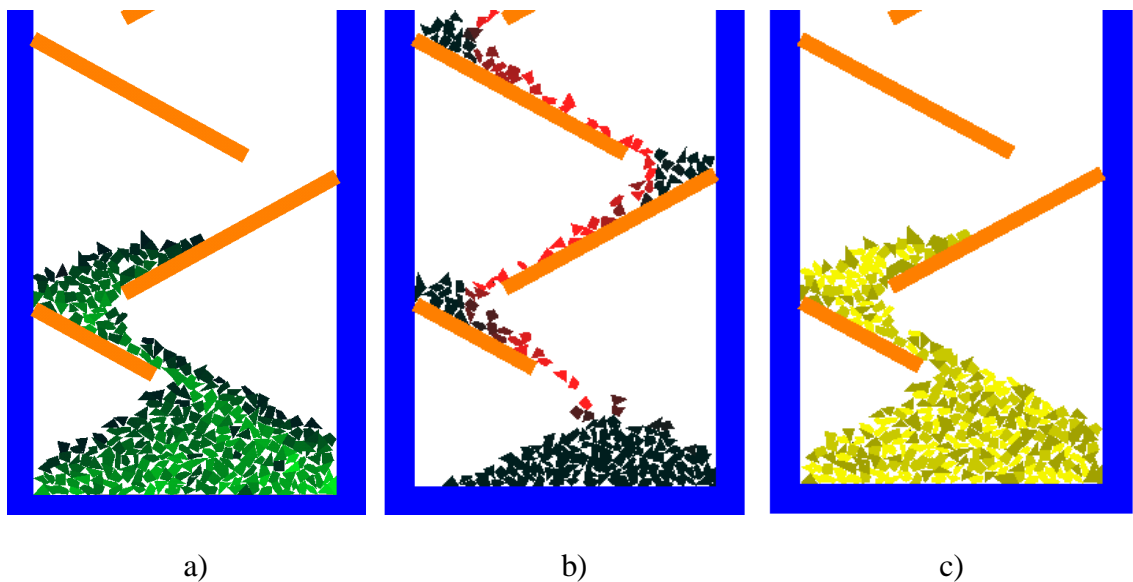
Figure 14.12. Cases of particles coloration depending on: a) force factor; b) velocity of center of mass; c) number of vertices.

14.4.2.3. XVA

In such tasks as a simulation of railway ballast where number of bodies can be very large, and equations of motion of the system are stiff, integration step is small enough, and CPU expenses for a single time step is large. Therefore such simulations take hours and even days. Calculated data (bodies coordinates, velocities, accelerations and so on) can be saved in XVA-file during integration of equations of motion of the system. This file very useful for creation of animation clips or viewing of modeled process in real time after simulation. There are two stages in work with XVA-files: file record and playing the file.

To record of a XVA-file a user perform the following actions:

- go to the **XVA** tab on **Object simulation inspector** dialog before starting simulation of an object;
- turn on **On** check box;
- specify the name of XVA-file (by default it is the model name).
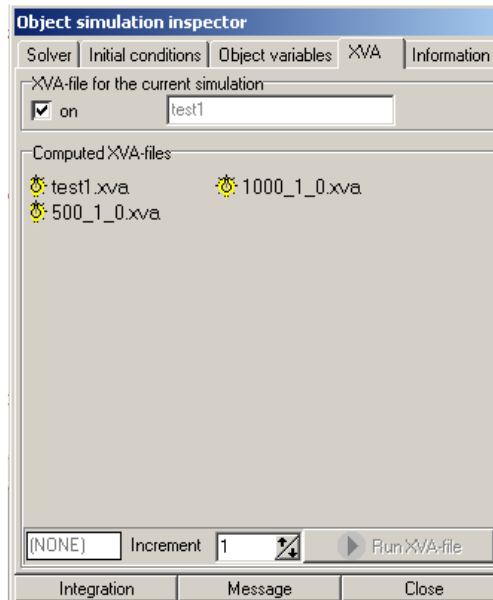
It is possible to play new XVA-file after process of integration completion. To play the XVA-file do the following actions:

- go to the **XVA** tab on **Object simulation inspector** dialog;
- click on a corresponding XVA-file by a mouse;
- click the **Run XVA-file** button.

XVA-file contains calculated data on every step of integration. As it was mentioned said earlier this time-step is very small. Using the **Increment** edit box allows to skip more steps of integration. So model time flows faster during playing XVA-file (than during simulation) and its speed can be even more then real-time speed. For example, value of increment 5 means that all steps except every fifth will be ignored.

14.5. Examples

14.5.1. Statistics of geometry of particles

Geometry parameters of particles statistics are available at **Geometry of particles** dialog, see Sect. 14.3.2.2.1. *"Geometry of particles"*, p. 14-15. This paragraph includes two examples of using **Geometry of particles** tool.

Example 1: Granular media consists of plastic regular pentagons with characteristic sizes 5, 10 and a 20 cm, with ratio 6:2:1.

- Open **Geometry of particles** dialog.
- Set **Number of types of polygon** edit box to **1**. It means that granular media consists only of one faction (one kind of polygons).
- Set empty cell of **Vertex count** row of fraction table to 5. It means that the system will consist of pentagons.

- Set empty cell of **Ratio** row of fraction table to **1**. Actually, the entered number can be any positive number, because number of types is only one.
- Set empty cell of **Density** row of fraction table to **1800**. This value corresponds plastic materials.
- Select **horizontal** in **Characteristic dimension** drop-down box.
- Set **Element count** to **3**. It means that there are three factions on horizontal characteristic sizes.
- Fill **Elements** table with 0**.05, 0.1** and **0.2** values.
- Select **vertical** in **Characteristic dimension** combo box.
- Set **Element count** to **3**. It means there are 6 fractions of characteristic sizes (on a picture below they are highlighted by red color).
- Fill **Elements** table with **0.05, 0.1** and **0.2** values.
- Fill diagonal of **Histogram of characteristic dimensions** with **6, 2** and **1**. It should look like on a figure below. As we set only 3 values on diagonal we get three factions on sizes: 5x5, 10x10 and 20x20 cm.
- Turn on **Convexity** check box.

Histogram of characteristic dimensions

	0.05	0.1	0.2
0.05	6	0	0
0.1	0	2	0
0.2	0	0	1

- Save the entered data by clicking 💾 button as test1.bst file.
- Click **OK** button.

The result of using such statistics of particle geometry[2] is granular media shown in figure below (after the process of filling).
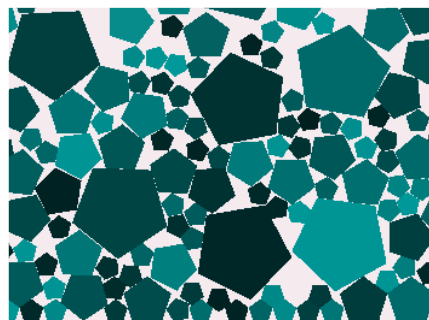
Example of 2: Ballast media consists of big square and small round particles in the ratio of 6 to 1, thus side of every square a 2 mm and diameter of circle are 0.57 mm. Granular media with a such fractional composition is used for the simulation of process of spontaneous stratification (passive layer sorting of particles depending by geometrical sizes and form).

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[2] Note that for creation of granular media it needs to set initial parameters of granular media, hard box parameters and so on.

It is assumed that a task is already created and opened in the simulation program, see Sect. 14.3. *"Creating models"*, p. 14-9.

- Open **Geometry of particles** dialog. For this purpose: open **Ballast parameters** dialog (the button 🔲 of **Ballast** panel of simulation program main form), select **Infill** tab and click the **Geometry of particles** button.

- Set **Number of types of polygon** edit box to **2**. It means that a granular media consists of two kinds of polygons: squares (4 tops) and heptagons (interpolated circles). There is a possibility to use **Circle** check box to create circles instead of heptagons.

- Fill **Number of vertices / Ratio** table as shown on a figure below. It means that quadrangular particles will be created six times less than heptagonal ones.
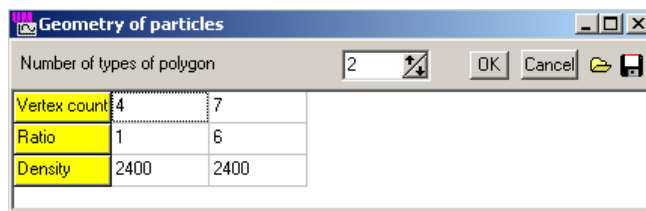
- Go to any cell of the second column of **Number of vertices / Ratio** table (with a value 4, 1, or 2400) so **Vertices count** text label in middle part of dialog was equal to **4** (white digit on red background). It means that now a user works with geometry of quadrangles.

- Select **horizontal** in **Characteristic dimension** combo box.

- Set single **Elements** table cell to **0.002**.

- Select **vertical** in **Characteristic dimension** combo box.

- Set single **Elements** table cell to **0.002**.

- **Histogram of characteristic dimension** should look like on a figure below. It means that quadrangular particles are squares with 2 mm side. If a user set **Irregularity** edit box to nonzero value, then quadrangles with characteristic sizes 2x2 mm will be created instead of squares. **Convexity** check box should be turned on.
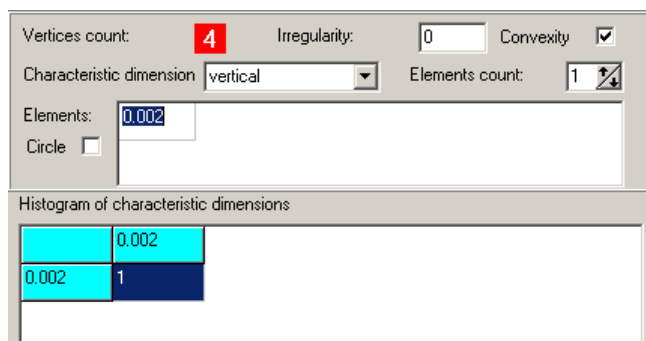

- Go to setting of characteristic sizes of septangles. For this purpose, click on any cell of the third column (values 7, 6 or 2400) of **Number of vertices / Ratio** table, **Vertices count** text label in middle part of dialog was equal to **7** (white on red).

- Select **horizontal** in **Characteristic dimension** combo box.

- Set single **Elements** table cell to **0.00057**.

- Select **vertical** in **Characteristic dimension** combo box.

- Set single **Elements** table cell to **0.00057**.

- **Histogram of characteristic dimension** should look like on a figure below. It means that heptagonal particles are regular septangles with 0.57 mm characteristic size along both directions. **Convexity** check box should be turned on.
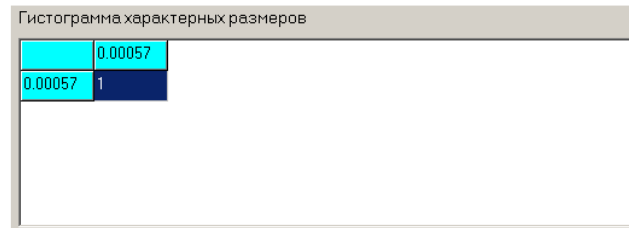
- Save the entered data by clicking ⊞ button in test2.bst file.
- Click **OK** button.

The result of using such statistics of particle geometry is granular media shown on figure below (before the process of filling).
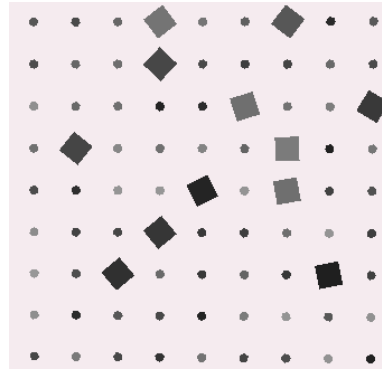
14.5.2. Samples of box geometry

Using different geometry of box parameters and parameters of initial position of a granular media allows to obtain the following cases of constraints:
- horizontal surface;
- horizontal surface with a vertical wall;
- box;
- volume defined by a user.

Few examples of different configuration of granular media relative to a hard box are described in this paragraph.

14.5.2.1. Horizontal base

- Open a model, containing a **Ballast** subsystem in the simulation program.
- Open **Ballast parameters** dialog by clicking ⊞ button of **Ballast** toolbar.
- Select **Box** tab and set parameters as indicated on a figure below.

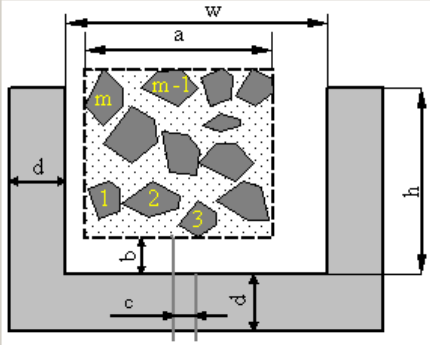| Width (w) | 100 |
Height (h)	2
Wall width (d)	0.2
Horizontal shift (c)	0

- Select **Infill** tab and set parameters as indicated on a figure below.

| Width (a) | 0.5 |
Initial height (b)	0.2
Space between particles	0
Vertical translation	0

- Open **Geometry of particles** dialog by clicking **Geometry of particles** button and load file of statistics of particles geometry *test1.bst* by ⌷ button. Click **OK** button to confirm the chosen of parameters from the file.

- Click ⟳ button for creation of granular media. The result should be look as shown on a figure below.

14.5.2.2. Horizontal base with left vertical wall

- Set **Horizontal shift** edit box on **Box** tab to **49.5** as opposite to horizontal base case.
- Press ⟳ button for creation of granular media. The result should be look as shown on a figure below.
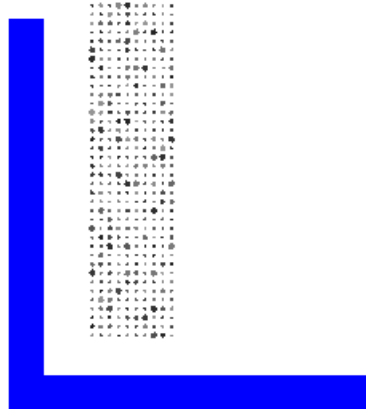
14.5.2.3. Box

- Set Width on Box tab to 1 as opposite to horizontal base case.
- Click ⟳ button for creation of granular media. The result should be look as shown on a figure below.
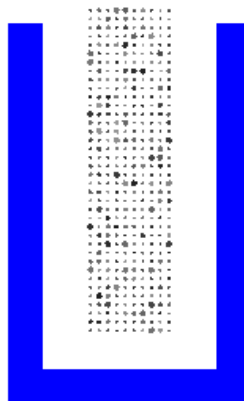
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