User`s manual

Simulation of MBS with Pneumatic Systems

2020

Contents

31. Simulation of multibody systems with pneumatic elements

31.1. General information

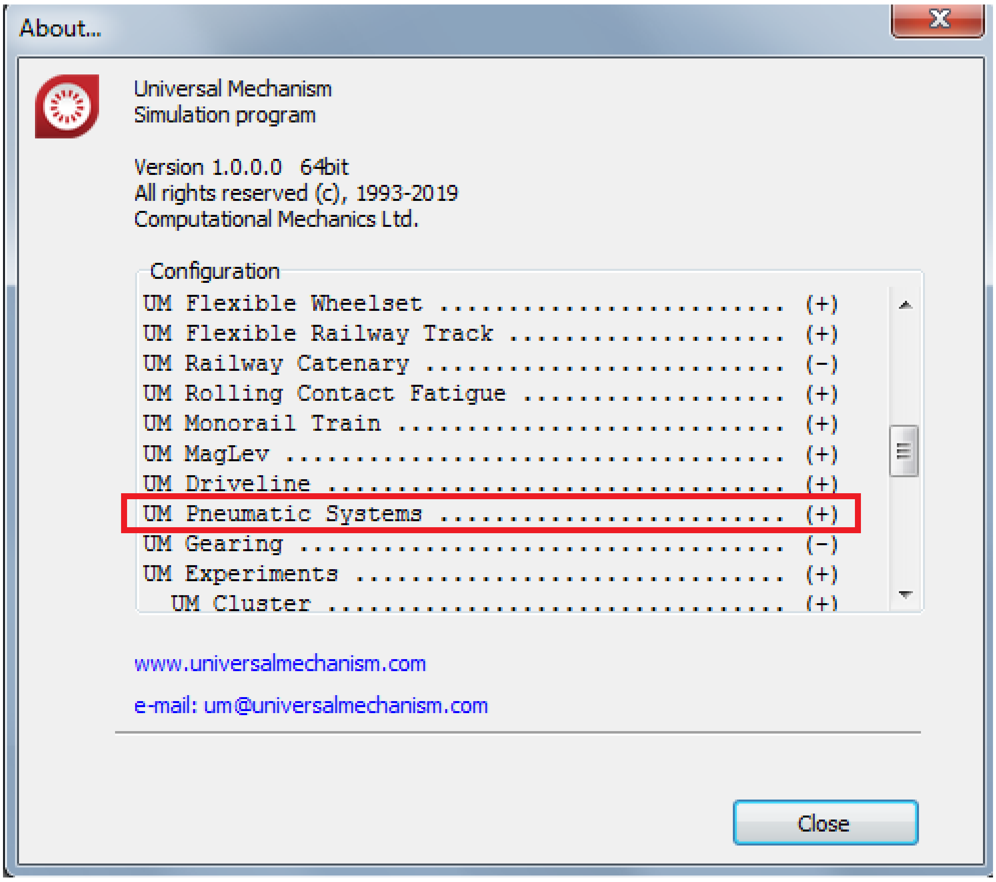


Figure 31.1. '**About**' window. List of available modules

Program package Universal Mechanism includes a specialized module **UM Pneumatic Systems** (UM PS), which contains tools for simulation of models with pneumatic elements, Figure 31.1. The following elements are available in the module:

- Air springs;
- Rigid chambers;
- Pneumatic lines;
- Orifices.

In this manual we use the following designations:

p - pressure (Pa);

V - volume (m^3);

T - temperature (K);

R= 287.058 - specific gas constant (J/(kg·K));

n - polytropic index;

m - mass (kg);

d, *D* - diameter;

A - area (m^2);

L - length (m).

Standard Reference Atmospheric conditions (ANR):

$$p_0 = 101.325 \text{ kPa},$$ (31.1)

$$T_0 = 293.15 \text{ K} = 20°\text{C},$$

$$AH_0 = 65\%$$

Air is considered as an ideal gas satisfying the law

$$pV = mRT.$$ (31.2)

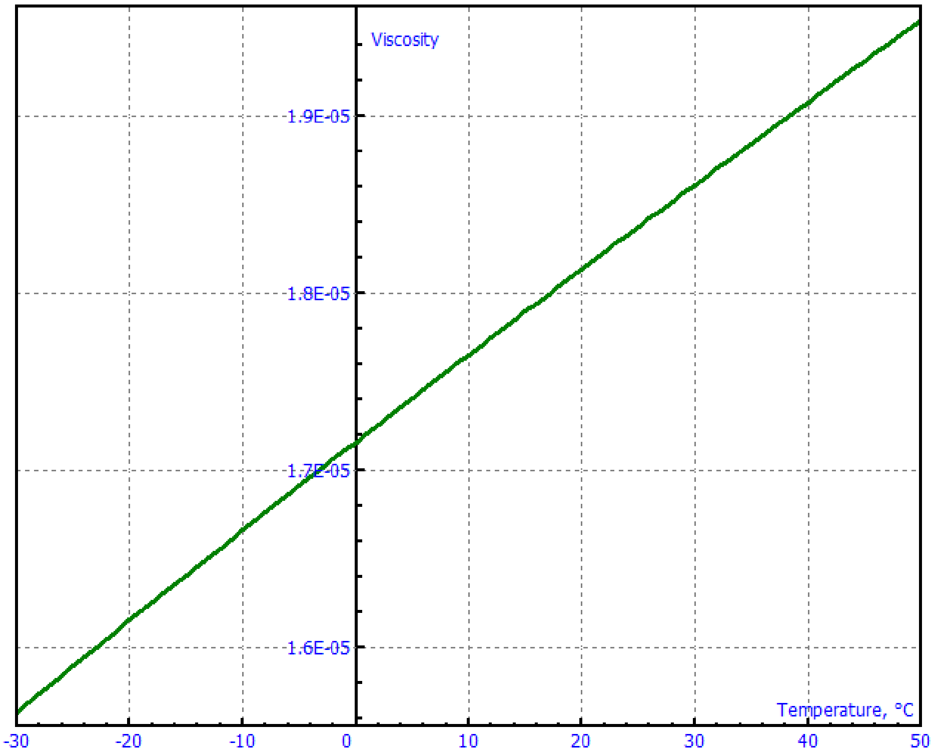


Figure 31.2. Dynamic viscosity of air versus temperature

Sutherland's law is used for evaluation of dynamic viscosity of air on temperature, Figure 31.2, [1]:

$$\mu = \mu_0 \frac{T_{ref}+S}{T+S}\left(\frac{T}{T_{ref}}\right)^{3/2},$$ (31.3)

Where μ_0=1.716kg/(ms), T_{ref}=273,15 K, S=110.4 K.

31.2. Pneumatic elements

Here we consider pneumatic elements, which are included in the module UM Pneumatic Systems.

31.2.1. Chambers

The chamber state is described by the polytropic thermodynamic process

$$p\left(\frac{V}{m}\right)^n = c = \text{const.} \tag{31.4}$$

Here p is the pressure, which is assumed to be the same inside the chamber, V is the chamber volume, and n is the polytropic index. The air mass in a chamber is computed as

$$m = m_0 + \sum_i \int_0^t \dot{m}_i \, dt,$$

where $\dot{m}_i$ is the mass flow rate of the connected line or orifice i, Sect. 31.2.5 *Pneumatic lines*, 31.2.6. *Orifices*.

Taking into account the ideal gas law (31.2), the temperature is computed as

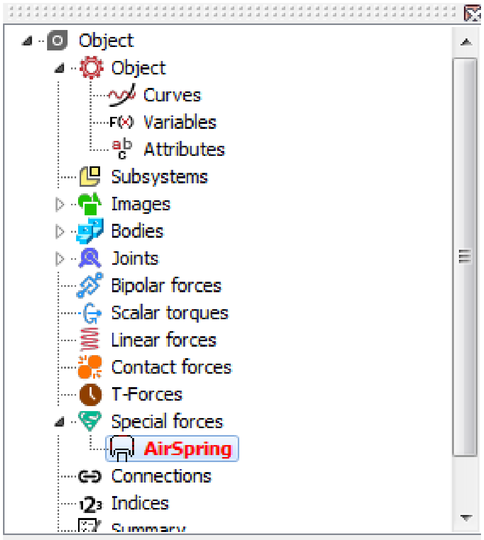
$$T = \frac{pV}{mR}.$$

31.2.2. Rigid chambers

This element corresponds to a chamber with a constant volume *V*=const and a variable air mass. The chamber pressure is computed directly from the equation of polytropic process (31.4)

$$p = cm^nV^{-n}.$$

31.2.3. Air springs



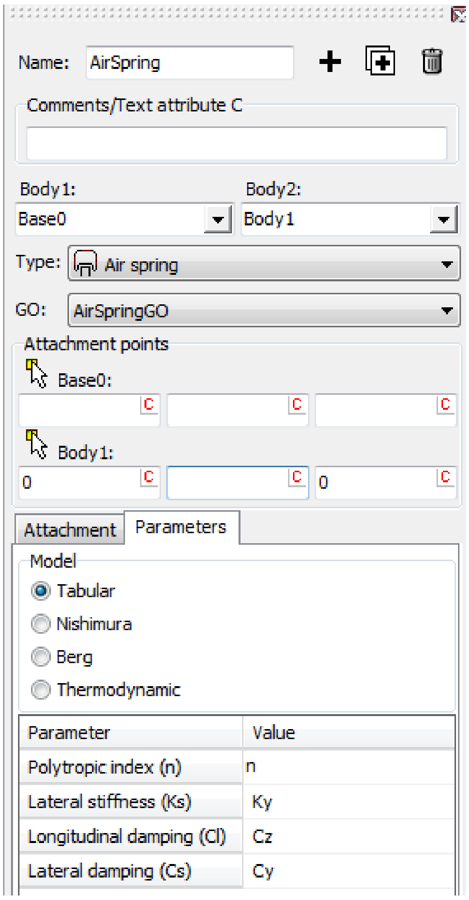


Figure 31.3. Air spring as a special force element

UM Pneumatic Systems includes advanced models of air springs (AS) as special force elements, Figure 31.3. The following air spring models are available in UM:

- **Tabular model**: the description of force element includes tabular experimental data on force and volume; this model is considered as the most exact one (included in UM PS)
- Nishimura model (included in UM Base)
- Berg model (included in UM Base)
- Thermodynamic model (included in UM Base)

It is important that only the **tabular model** of AS is included in UM PS module and describes in this chapter below. In particular, the tabular AS can be connected by pneumatic lines.

31.2.3.1. Tabular model of air spring

Manufacturers often supply information about AS properties in the form of a static data charts, which include load/height and volume/height diagrams. Examples of such charts for 1T15-M0 [2] and Numatics ASNS10-2-1 [3] are shown in Figure 31.4. Such data allow development of rather exact mathematical models of AS as it is described below.

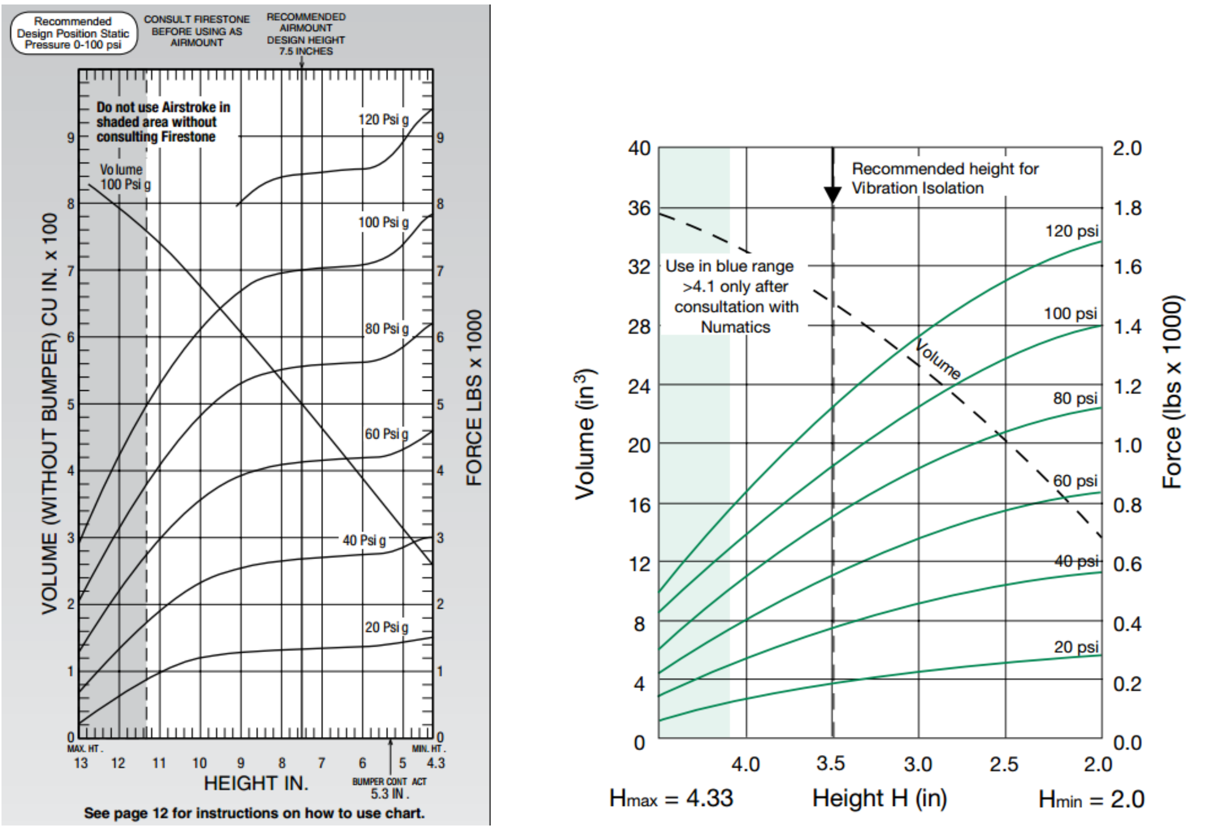


Figure 31.4. Force and volume versus deflection for air spring 1T15-M0 (left) and Numatics ASNS10-2-1

31.2.3.1.1. Parameters of tabular air spring in Input program

Tabular AS description in Input program requires specifying the following list of parameters (Figure 31.3):

- polytropic index; the default and recommended value for AS is n=1.38, see [4], [5];
- lateral stiffness and damping constants as well as the longitudinal damping constant.

These data can be parameterized by identifiers.

Tabular description of AS should be done in Simulation program, Sect. 31.2.3.1.3.4. *Creating UM files *.ast with tabular air spring models.*

31.2.3.1.2. Tabular data format

H \ P	20	40	60	80	100
9.5	2537.2	4782.7	7035	9410.5	11966
10	2385.2	4553.9	6792.5	9140.6	11579
10.5	2246.8	4467.6	6677.4	8966.7	11346
11	2233.1	4379.9	6511.6	8733.8	11061
11.5	2194.7	4251.1	6336.3	8498.1	10775
12	2119.4	4134.7	6165	8274.8	10496
12.5	2048.2	4019.6	6025.3	8106.3	10276
13	1983.8	3936	5929.4	8004.9	10128
13.5	1938.6	3886.7	5880	7951.5	10059
14	1898.8	3862	5859.5	7922.7	10027
14.5	1886.5	3851.1	5832.1	7900.8	10006
15	1874.2	3845.6	5819.8	7876.1	9984.6
15.5	1870.1	3827.8	5804.7	7869.3	9957.2
16	1863.2	3812.7	5788.3	7850.1	9947.6

a)

H \ P	20	40	60	80	100
9.5	775.42	837.07	898.72	960.37	1022
10	809.67	879.54	949.41	1019.3	1089.2
10.5	905.57	967.22	1028.9	1090.5	1152.2
11	954.89	1022	1089.2	1156.3	1223.4
11.5	1007	1078.2	1149.4	1220.7	1291.9
12	1050.8	1126.1	1201.5	1276.8	1352.2
12.5	1098.7	1176.8	1254.9	1333	1411.1
13	1149.4	1228.9	1308.3	1387.8	1467.3
13.5	1219.3	1294.7	1370	1445.4	1520.7
14	1249.4	1331.6	1413.8	1496	1578.2
14.5	1304.2	1386.4	1468.6	1550.8	1633
15	1328.9	1419.3	1509.7	1600.2	1690.6
15.5	1390.6	1478.2	1565.9	1653.6	1741.3
16	1448.1	1534.4	1620.7	1707	1793.3

b)

H \ P	100
9.5	1022
10	1089.2
10.5	1152.2
11	1223.4
11.5	1291.9
12	1352.2
12.5	1411.1
13	1467.3
13.5	1520.7
14	1578.2
14.5	1633
15	1690.6
15.5	1741.3
16	1793.3

c)

Force Volume

Figure 31.5. Tabular data for air spring in Imperial units (fragment): force (a) and volume (b,c)

Tables. This type of AS description requires two tables: force/height and volume/height for different values of pressure like in Figure 31.5. Data should be ordered in the growth of the height (H) and pressure (P). Table columns correspond to the pressure. Volume can be given for one pressure (Figure 31.5 c), whereas the force data should correspond to the entire region of AS operation.

Other requirements:

- The force table must include data for at least two pressure and two height values.
- The force value must increase with the growth of the pressure.
- The volume table must include data for at least one pressure and two height values.
- The volume value must increase with the growth of both the pressure and the height.

Units. Data can be prepared both in SI and Imperial units, Table 1.

Table 1. Table data units

System of units	Height	Force	Pressure	Volume
SI	m	N	Pa	m^3
Imperial	in	lbf	psi g, psi a	in^3

Pressure type. Both absolute and gauge pressure data can be used.

31.2.3.1.3. Preparing and input tabular data

31.2.3.1.3.1. Chart digitizing

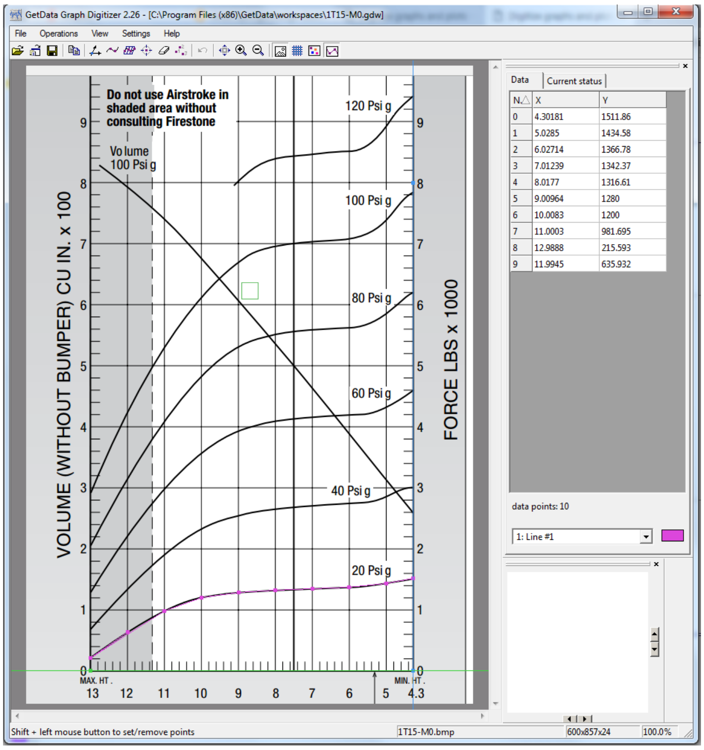


Figure 31.6. Digitizing plots for air spring 1T15-M0

If AS data are available as a chart like in Figure 31.4, software for plot digitizing should be used. We use the GetData Graph Digitizer software, Figure 31.6, [6].

31.2.3.1.3.2. Data preparing in Microsoft Excel

We recommend using the Microsoft Excel for preparing data according to the format and unit requirements. Data units can be easily changed with this tool to the desired ones; for example liters should be changed to m^3 and bars to Pa.

D	E	F	G	H	I	J	K	L
	20	40	60	80	100			100
4.3	1511.86	3008.81	4602.03	6203.39	7849.49		4.3	258.847
5	1434.58	2855.59	4317.29	5854.92	7384.41		5	313.627
6	1366.78	2744.41	4191.19	5617.63	7080.68		6	388.746
7	1342.37	2703.73	4157.29	5589.15	7033.22		7	463.322
8	1316.61	2649.49	4092.2	5509.15	6953.22		8	536.407
9	1280	2546.44	3926.78	5303.05	6692.88		9	607.051
10	1200	2328.14	3564.75	4823.05	6116.61		10	676.339
11	981.695	1901.02	2977.63	4082.71	5297.63		11	741.017
12	635.932	1334.24	2215.59	3148.47	4234.58		12	792.949
13	215.593	684.746	1290.85	2056.95	2909.83		13	838.78

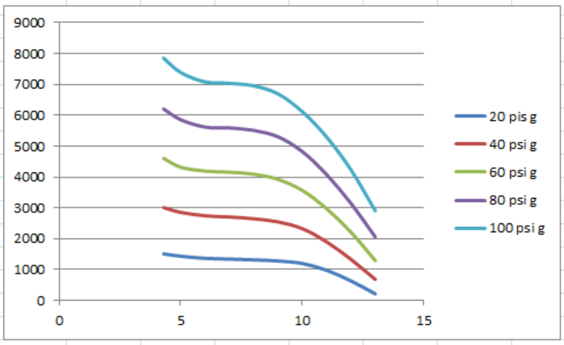
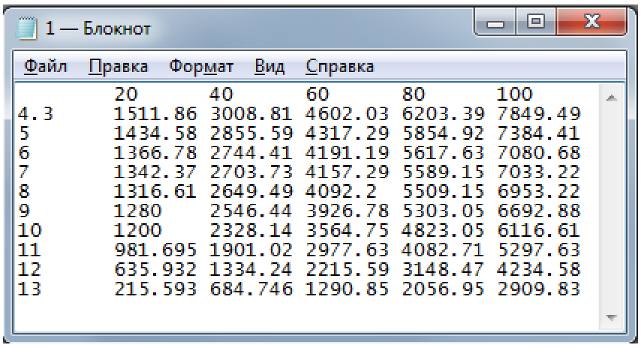


Figure 31.7. Force and volume data for 1T15-M0 in Microsoft Excel

31.2.3.1.3.3. Data preparing in text file



	20	40	60	80	100
4.3	1511.86	3008.81	4602.03	6203.39	7849.49
5	1434.58	2855.59	4317.29	5854.92	7384.41
6	1366.78	2744.41	4191.19	5617.63	7080.68
7	1342.37	2703.73	4157.29	5589.15	7033.22
8	1316.61	2649.49	4092.2	5509.15	6953.22
9	1280	2546.44	3926.78	5303.05	6692.88
10	1200	2328.14	3564.75	4823.05	6116.61
11	981.695	1901.02	2977.63	4082.71	5297.63
12	635.932	1334.24	2215.59	3148.47	4234.58
13	215.593	684.746	1290.85	2056.95	2909.83

Figure 31.8. Force data for 1T15-M0 in a text file

Force and volume data can be prepared in a text file, Figure 31.8. Numbers in the text should be separated by blanks or Tab symbols.

31.2.3.1.3.4. Creating UM files *.ast with tabular air spring models

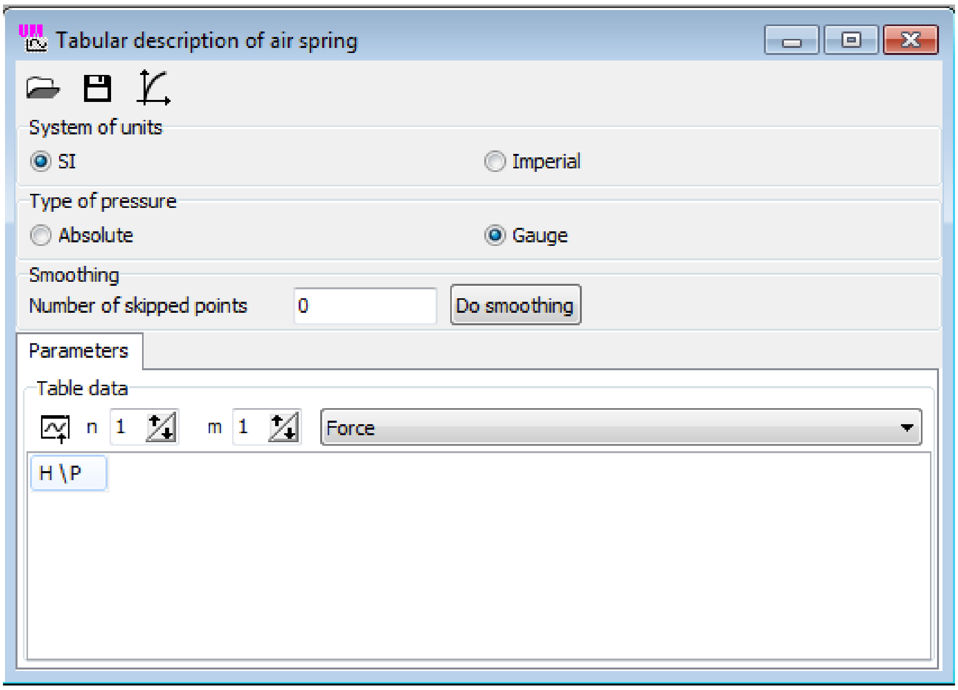


Figure 31.9. Window for creating a file with tabular air spring model

Tabular data with an AS model should be saved as a *.ast file. The default location of these files is the directory **{UM data}\AirSpring**, for example,

c:\Users\Public\Documents\UM Software Lab\Universal Mechanism\9\AirSpring\

To create a file

- Run UM Simulation program;
- Click on the **Tools | Tables for air springs…** command of the main menu to open the window *Tabular description of air spring*, Figure 31.9;
- If necessary, change

  - System of units

- Pressure type

- Select the type of data **Force** or **Volume**,
- Copy tabular data from the Excel or the text file:

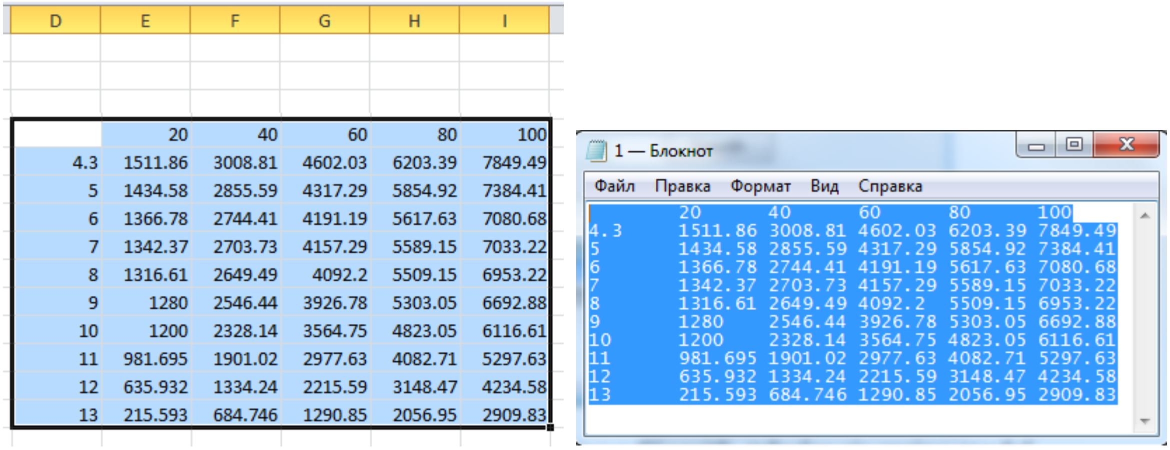
- Select the data including the first row with pressure values, Figure 31.10;

- Copy data in the clipboard by Ctrl+C;

- Click by the mouse on the empty area with tabular data in the window *Tabular description of air spring* Figure 31.9 to make the area active;

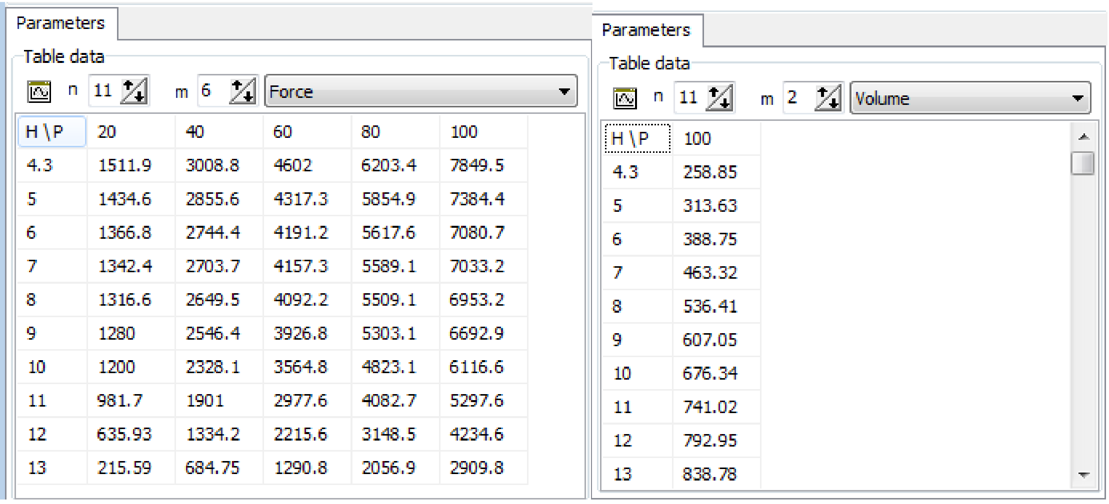
- Paste the data from the clipboard by Ctrl+V.

- After entering, verifying and possible modification data for the force and volume, save data to an *.ast file by the button.



	20	40	60	80	100
4.3	1511.86	3008.81	4602.03	6203.39	7849.49
5	1434.58	2855.59	4317.29	5854.92	7384.41
6	1366.78	2744.41	4191.19	5617.63	7080.68
7	1342.37	2703.73	4157.29	5589.15	7033.22
8	1316.61	2649.49	4092.2	5509.15	6953.22
9	1280	2546.44	3926.78	5303.05	6692.88
10	1200	2328.14	3564.75	4823.05	6116.61
11	981.695	1901.02	2977.63	4082.71	5297.63
12	635.932	1334.24	2215.59	3148.47	4234.58
13	215.593	684.746	1290.85	2056.95	2909.83

Figure 31.10. Selecting data in Microsoft Excel and in text editor



H \ P	20	40	60	80	100
4.3	1511.9	3008.8	4602	6203.4	7849.5
5	1434.6	2855.6	4317.3	5854.9	7384.4
6	1366.8	2744.4	4191.2	5617.6	7080.7
7	1342.4	2703.7	4157.3	5589.1	7033.2
8	1316.6	2649.5	4092.2	5509.1	6953.2
9	1280	2546.4	3926.8	5303.1	6692.9
10	1200	2328.1	3564.8	4823.1	6116.6
11	981.7	1901	2977.6	4082.7	5297.6
12	635.93	1334.2	2215.6	3148.5	4234.6
13	215.59	684.75	1290.8	2056.9	2909.8

H \ P	100
4.3	258.85
5	313.63
6	388.75
7	463.32
8	536.41
9	607.05
10	676.34
11	741.02
12	792.95
13	838.78

Figure 31.11. Ready air spring data for 1T15-M0 in Imperial system

Smoothing tabular data

Click on the **Smooth** button in Figure 31.9 one or several times for smoothing with a B-spline the experimental tabular data in columns, Figure 31.12. Skipping points allows a more cardinal smoothing of the experimental data.

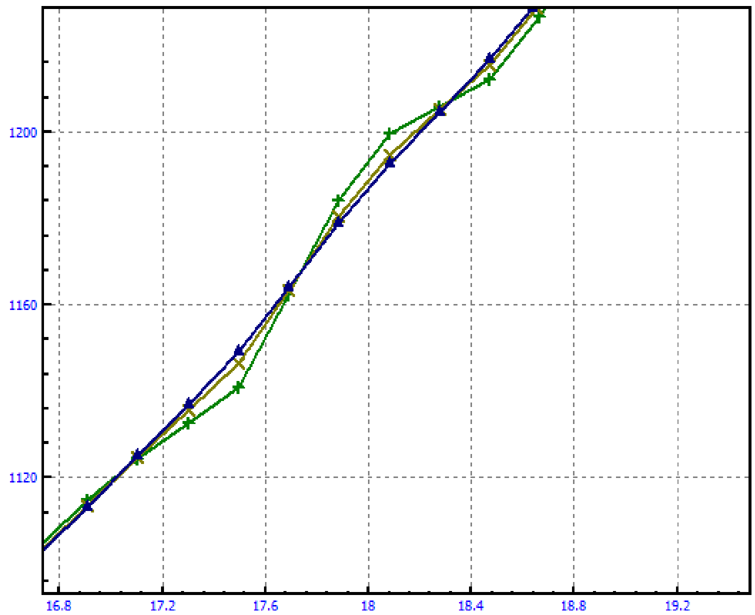


Figure 31.12. Single and double smoothing the tabular data

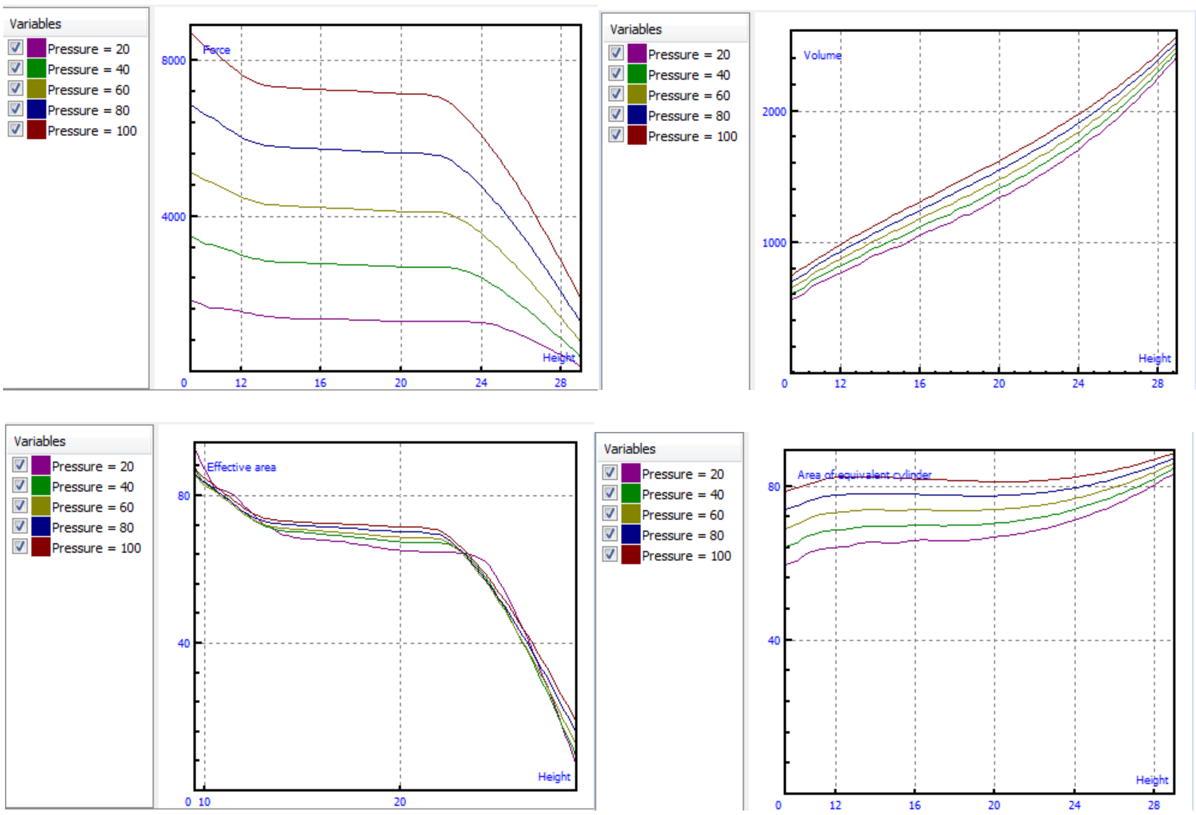


Figure 31.13. Plots for file 'Test data.ast'

Data plots

Use the  button in Figure 31.9 to get plots (see Figure 31.13)

- force F,
- volume V,
- effective area $A = F/p$,
- area of equivalent cylinder $A_c = V/h$.

31.2.3.1.3.5. Creating tabular data by effective area

Air spring description is often available as a dependence of the effective area on the height. Click on the button in Figure 31.9 to open the window for generation of tabular data by the charts of the effective area and volume verses height, Figure 31.14.

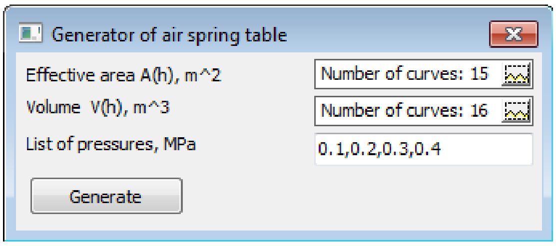


Figure 31.14. Window for input of effective area and volume

Both the effective area and the volume data are entered by the button. List of tabular values of the pressure in the increasing order should be specified as well. Figure 31.16 shows the curves corresponding to the AS description in paper [7].

After input of necessary data

- curve: effective area vs. height,
- curve: AS volume vs. height,
- list of tabular pressure values,

click on the **Generate** button to create the tables for the force and volume, Figure 31.17.

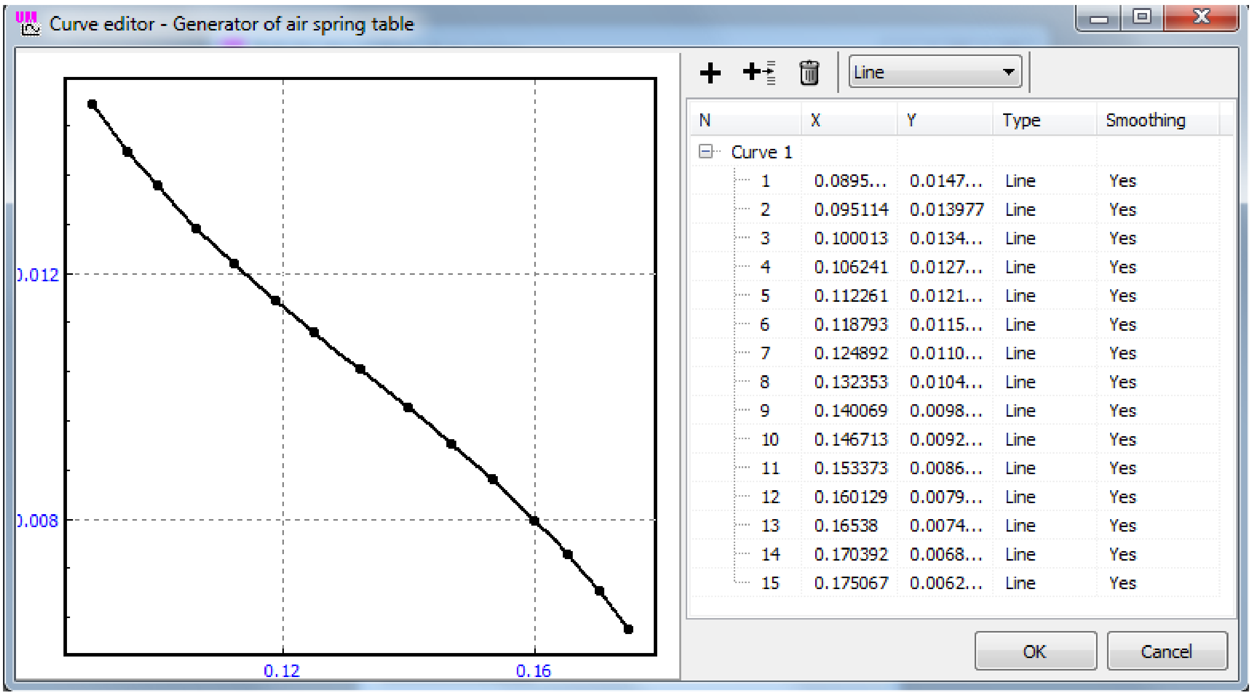


Figure 31.15. Effective area versus height

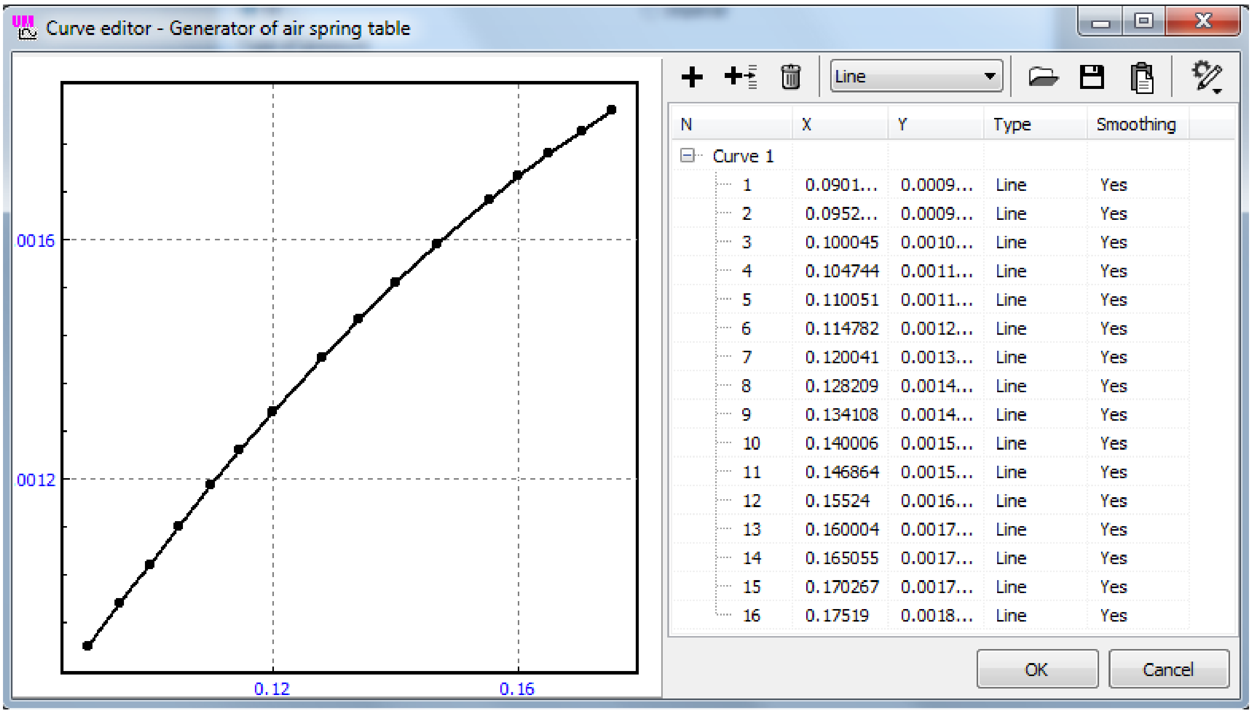


Figure 31.16. Volume versus height

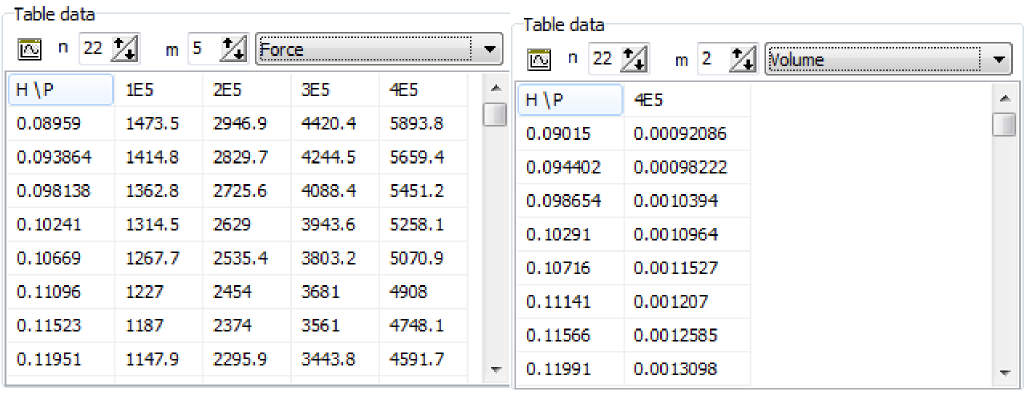


Figure 31.17. Generated tabular data

31.2.3.1.4. Mathematical model of air spring by tabular description

This model of AS uses experimental **static tabular data** on dependence of force F and the air bag volume V on the spring height h and air pressure p, which correspond to the isobaric change of the AS height, Figure 31.13.

$$F_s = F_s(p, h), \quad V_s = V_s(p, h). \tag{31.5}$$

UM uses both polynomial, in particular linear, and spline interpolation or extrapolation of the tabular data to get continuous functions (31.5) over the whole operation region of height and pressure.

A **dynamic load model** is the dependence of the force, pressure on height and volume in the case of the polytropic compression or depression of the AS bellow during change of its height. The model is computed for definite static load F_0 and height h_0, which can correspond to the equilibrium position of the UM object. Air mass inside the AS bellow is considered as constant, i.e. this is the model of an isolated AS.

$$F_d = F_d(h, F_0, h_0), V_d = V_d(h, F_0, h_0), p_d = p_d(h, F_0, h_0). \quad (31.6)$$

Let us consider an algorithm for computation of dynamic characteristics of AS (31.6).

- Compute static pressure p_0 from the nonlinear equation, which is the first of Eq. (31.5): $F_0 = F_s(p, h_0)$.
- Compute static volume from the second of Eq. (31.5): $V_0 = V_s(p_0, h_0)$.
- Compute static air mass from the law of the ideal gas for the given value of the temperature of environment T_e: $m_0 = RT_e/(p_0V_0)$.
- Compute polytropic constant from Eq. (31.4): $c = p_0\left(\frac{V_0}{m_0}\right)^n$.
- For each value of the height h compute V_d, p_d from the system of two nonlinear equations for the polytropic process (31.4) and the second of Eq. (31.5). Compute the force F_d from the first of Eq. (31.5).

This algorithms is applied to computation of AS force in the case of connection of the AS bellow with other chambers by pneumatic lines and orifices. The only difference consists in the variation of air mass inside the AS bellow, which is computed as

$$m = m_0 + \sum_i \int_0^t \dot{m}_i \, dt,$$

where $\dot{m}_i$ is the mass flow rate of the connected line or orifice i, Sect. 31.2.5 *Pneumatic lines*, 31.2.6. *Orifices*.

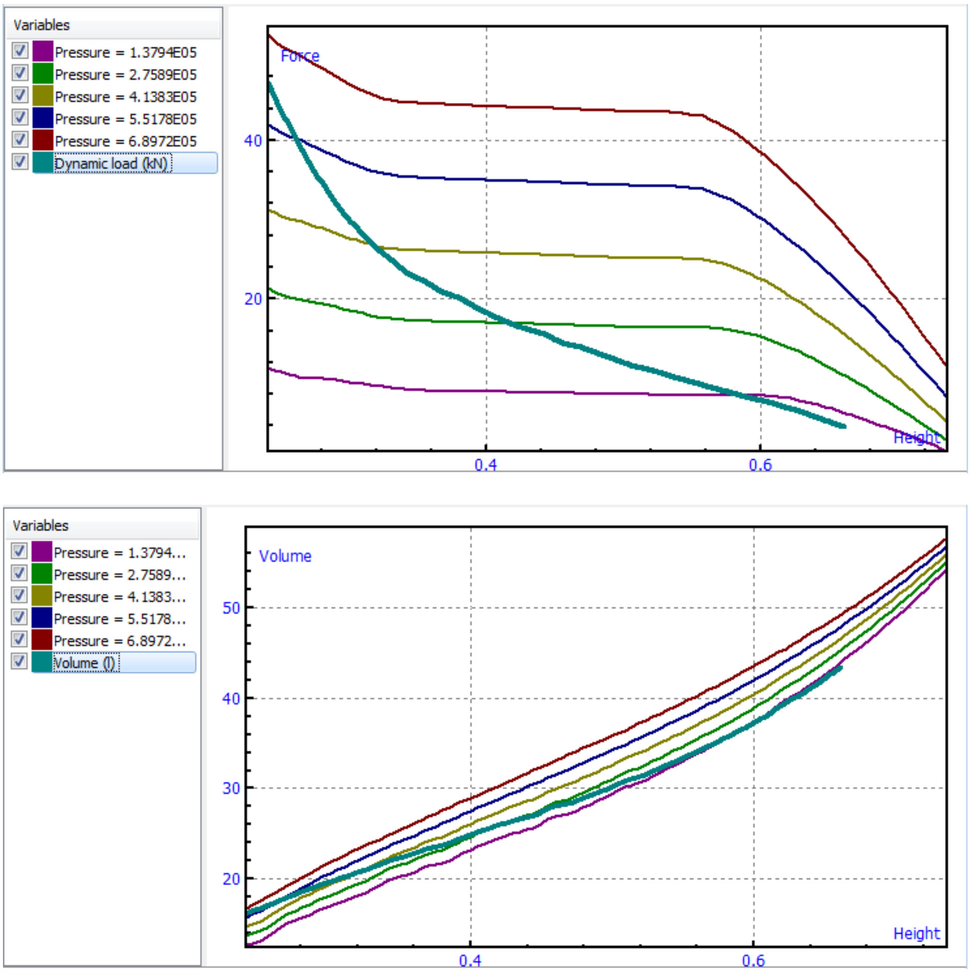


Figure 31.18. Force and volume for static and dynamic load, file 'Test data.ast'

Force and volume versus height for static and dynamic load are compared in Figure 31.18. Plots for dynamics load are drawn by thick lines. Results are computed for n = 1.38, $F_0 = 12$kN, h_0 = 0.5m.

Air spring specifications contain often a dependence of the volume on the height for one pressure only, Figure 31.7. Therefore, it is possible to use the simplified version of Eq. (31.5), when the dependence of the volume on the pressure is ignored:

$$F_s = F_s(p,h),\ \ V_s = V_s(h). \tag{31.7}$$

The above algorithm is easily applied to this simplified case.

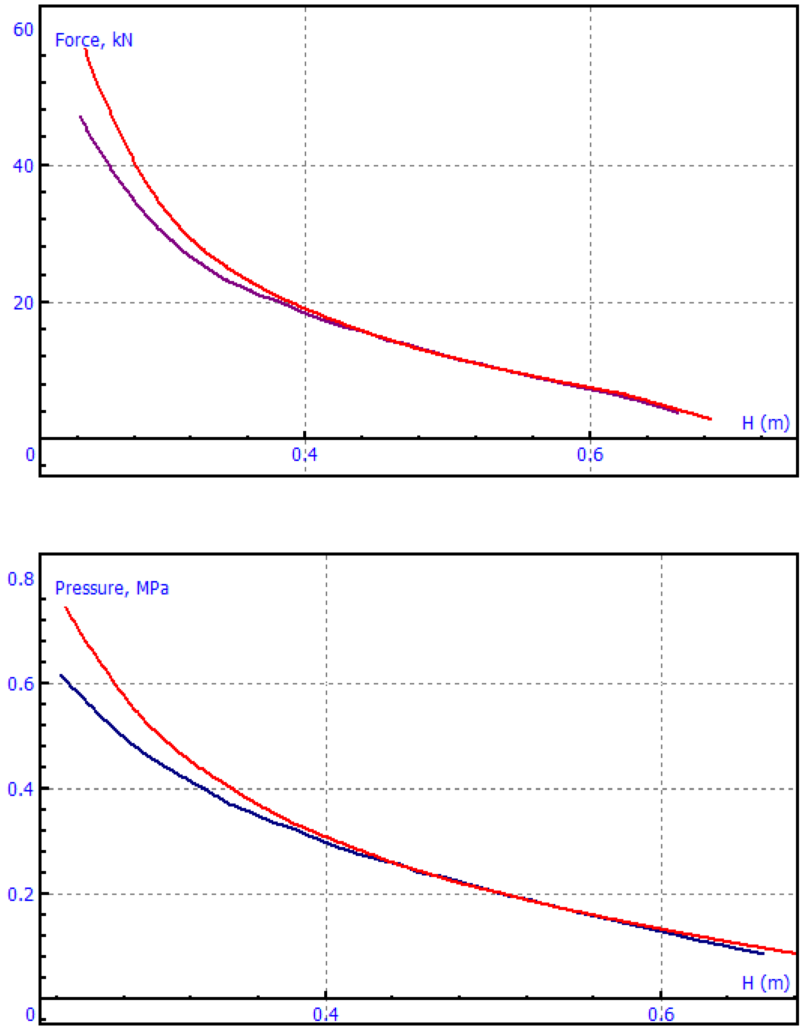


Figure 31.19. Comparison of force and pressure vs. height for dynamic load: full and simplified (red) models

Solutions for dynamic load in the case of the full AS tabular model (file 'Test data.ast') with a simplified model, where the force/height static load model is the same, but volume/height curve is used for one (the maximal) pressure only (file 'Test data 2.ast'). This example shows that the simplified model has results close to the full one in the region of heights near the static value h_0. The maximal deviation about 20% is observed for the low AS heights.

31.2.3.1.5. Verification of mathematical model

Compare mathematical model described in the previous section with the standard engineering calculation of dynamic load according to [5], [4]. Consider AS 1T15-M0 as an example. The dynamic load is computed in UM according to the algorithms described in the previous section for static load F_0 =18kN and static height $h_0 =$ 0.2032m. Eq. (31.4) with polytropic index n=1.38 and a spline interpolation of tabular data form Figure 31.11 converted to SI (Figure 31.21) for Eq. (31.7) are used for dynamic load computation.

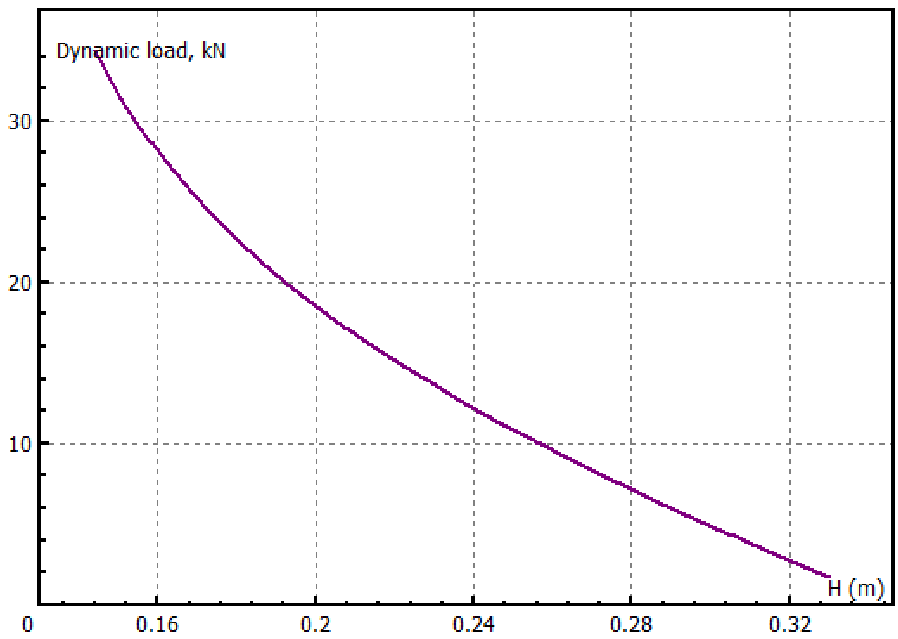


Figure 31.20. Dynamic load vs. height for AS 1T15-M0

H \ P	1.3794E5	2.7589E5	4.1383E5	5.5178E5	6.8972E5
0.10922	6727.5	13389	20478	27604	34929
0.127	6383.6	12707	19211	26053	32859
0.1524	6081.9	12212	18650	24997	31508
0.1778	5973.3	12031	18499	24871	31297
0.2032	5858.7	11790	18210	24515	30941
0.2286	5695.8	11331	17473	23598	29782
0.254	5339.8	10360	15862	21462	27218
0.2794	4368.4	8459.2	13250	18167	23573
0.3048	2829.8	5937.1	9859	14010	18843
0.3302	959.35	3047	5744	9153	12948

H \ P	6.8972E5
0.10922	0.0042417
0.127	0.0051394
0.1524	0.0063704
0.1778	0.0075925
0.2032	0.0087901
0.2286	0.0099478
0.254	0.011083
0.2794	0.012143
0.3048	0.012994
0.3302	0.013745

Force (N) Volume (m^3)

Figure 31.21. Data for 1T15-M0 in SI

Let us compute the dynamic load for two AS heights h_1 =0.1524m and h_2 =0.254m following the detailed instructions in design guide [5].

- Volume at static position from Figure 31.21: V_0 =0.0087901m^3;
- Force at $h_0 =$ 0.2032m from the third column in Figure 31.21: F_{r0} =18210N;

- Pressure at $h_0 = 0.2032$m from the third column in Figure 31.21 p_r =413830Pa;
- Effective area at static position $A_0 = F_{r0}/p_r$= 0.0440036m^2;
- Gauge pressure at static position $p_{0g} = F_0/A_0$=409058Pa;
- Absolute pressure at static position $p_{0a} = p_{0g} + 101325$ =510383Pa;
- Volume at position 1 from Figure 31.21: V_1 =0.0063704m^3;
- Force at position 1 from the third column in Figure 31.21: F_{r1} =18650N;
- Effective area at position 1: $A_1 = F_{r1}/p_r$= 0.0450668m^2;
- Absolute pressure at position 1 from Eq. (31.4): $p_{1a} = p_{0a}(V_0/V_1)^{1.38}$=795898Pa;
- Gauge pressure at position 1: $p_{1g} = p_{1a} - 101325$ =694573Pa;
- Dynamic load at position 1: $F_1 = p_{1g}A_1$ =31302N;
- Volume at position 2 from Figure 31.21: V_1 =0.011083m^3;
- Force at position 2 from the third column in Figure 31.21: F_{r2} =15862N;
- Effective area at position 2: $A_2 = F_{r2}/p_r$= 0.0383297m^2;
- Absolute pressure at position 2 from Eq. (31.4): $p_{1a} = p_{0a}(V_0/V_2)^{1.38}$=370664Pa;
- Gauge pressure at position 2: $p_{2g} = p_{2a} - 101325$ =269339Pa;
- Dynamic load at position 2: $F_2 = p_{2g}A_2$ =10324N;

Now we get load values from UM graphic window by mouse picking:

$$F_{1UM} = 31756\text{N},\ F_{2UM} = 10125\text{N},$$

which gives the deviation 1.4% and 1.9% from the above values F_1, F_2.

31.2.4. Simple nodes

A simple node is a connection of any number of pneumatic lines. The simple node has two state variables: pressure and temperature. Let *i* be the index of a simple node and $\dot{m}_{ij}$ are the mass flow rate of line *l* connected to the node. The mass flow rate is positive $\dot{m}_{il} > 0$, if the flow comes into the node, i.e. the node pressure p_i is lower than the pressure of another chamber or node adjusted to the line. The mathematical model of the node corresponds to the Kirchhoff's law

$$\sum_l \dot{m}_{il} = 0. \qquad (31.8)$$

31.2.5. Pneumatic lines

A line connects two nodes *i*,*j*, each of them is a chamber or a simple node. The line length is *L*, the diameter is *d* or *D*.

A *stationary* line model includes a dependence of the mass flow rate on the pressure drop

$$\Delta p = p_1 - p_2 \qquad (31.9)$$

where $p_1 = \max(p_1, p_2)$, $p_2 = \min(p_1, p_2)$.

A *time domain* line model is described by a system of partial differential equations for the pressure and the mass flow rate

$$p = p(x,t), \dot{m} = \dot{m}(x,t),$$

where *x* is the longitudinal coordinate along the line.

Below we consider both stationary and dynamic models for the line, which are implemented in UM.

31.2.5.1. Stationary pipeline models

31.2.5.1.1. Mass flow rate model "Atlas"

The model is based on the ISO 6358 nozzle model for the flow rate [8]

$$\dot{m} = \begin{cases} p_1 C \rho_0 \sqrt{\dfrac{T_0}{T_1}} \sqrt{1 - \left(\dfrac{\frac{p_2}{p_1} - b}{1 - b} \right)^2}, & \dfrac{p_2}{p_1} > b \\ p_1 C \rho_0 \sqrt{\dfrac{T_0}{T_1}}, & \dfrac{p_2}{p_1} \le b \end{cases} \tag{31.10}$$

where C is the sonic conductance, b is the critical pressure ratio, T_1 the temperature in the node 1 (K), and ρ_0, T_0 are the reference density (kg/m^3) and temperature (K) (31.1).

The "Atlas" model [9] includes empirical formulas for the C, b parameters

$$C = \frac{0.029\mathrm{D}^2}{\sqrt{\frac{L}{D^{1.25}} + 510}}, b = \frac{474C}{D^2}. \tag{31.11}$$

It is known a disadvantage of the ISO 6358 model from the computation point of view: the derivative of the mass flow rate tends to infinity when the pressure drop goes to zero,

$$\frac{d\dot{m}}{d\Delta p} \xrightarrow[\Delta p \to 0]{} \infty. \tag{31.12}$$

This fact results in problems for numerical methods. Following to suggestion in [8], we replace the model (31.10) with the linear one in the region of the laminar flow. Let us derive the critical value of the ratio $r_b = p_2/p_1$ corresponding to Re*=2000.

$$\dot{m}^* = A\rho_1 w^* = \frac{A\mu \mathrm{Re}^*}{D} = p_1 C \rho_0 \sqrt{\frac{T_0}{T_1}} \sqrt{1 - \left(\frac{r_b^* - b}{1 - b} \right)^2} \approx p_1 C \rho_0 \sqrt{1 - \left(\frac{r_b^* - b}{1 - b} \right)^2},$$

$$r_b^* = b + (1 - b)\sqrt{1 - \left(\frac{A\mu \mathrm{Re}^*}{DC\rho_1} \right)^2} \approx 1 - 0.5(1 - b)\left(\frac{A\mu \mathrm{Re}^*}{Dp_1 C \rho_0} \right)^2.$$

Here A is the line section area. Consider an example: for a line with L=10m, D=5mm, $p_1 = p_0$ we obtain $r_b^* \approx 0.991$. The laminar flow model is

$$\dot{m} = \dot{m}^* \frac{1 - r_b}{1 - r_b^*}, \qquad r_b = \frac{p_2}{p_1} > r_b^*. \tag{31.13}$$

31.2.5.1.2. Mass flow rate model "Fluid mechanics"

This model is based on the fluid energy equation, [10]

$$\ln \frac{w_1}{w_2} + \frac{p_2^2 - p_1^2}{2p_1 \rho_1 w_1^2} + \frac{\lambda(w_1)L}{2D} = 0 \tag{31.14}$$

where w is the flow velocity, and $\lambda(w_1)$ is the friction factor, depending on the Reynolds number

$$\mathrm{Re} = \frac{\rho_1 D w_1}{\mu}.$$

For the laminar flow [8]

$$\lambda = \frac{64}{\mathrm{Re}}, \mathrm{Re} < 2000. \tag{31.15}$$

In the case of a turbulent flow, the Blasius equation is used

$$\lambda = \frac{0.3164}{\mathrm{Re}^{0.25}}, \mathrm{Re} > 4000 \tag{31.16}$$

or the Nikuradse–Prandtl–Karman (NPK) implicit equation

$$\frac{1}{\sqrt{\lambda}} = 2\log\left(\mathrm{Re}\sqrt{\lambda}\right) - 0.8. \tag{31.17}$$

We use the linear interpolation of the friction factor between the boundary values computed according to Eq. (31.15) and (31.16) or (31.17) for $\mathrm{Re} \in [2000, 4000]$.

In addition, we assume that

$$\frac{w_1}{w_2} \approx \frac{p_1}{p_2}$$

and Eq. (31.14) can be rewritten as

$$w_1 = \sqrt{\frac{p_1^2 - p_2^2}{2p_1\rho_1\left(\frac{f(w_1)L}{2D} + \ln\frac{p_1}{p_2}\right)}} \tag{31.18}$$

Eq. (31.18) is the nonlinear function of the flow velocity w_1. It can be solved by direct iterations.

The mass flow rate is then computed as

$$\dot{m} = \mathrm{A}\rho_1 w_1 = \frac{\pi D^2}{4}\rho_1 w_1 \tag{31.19}$$

31.2.5.1.3. Darcy-Weisbach equation

The Darcy-Weisbach equation [11] is a relation between the head loss h in a pipe and the average flow velocity *w*

$$h = \frac{(p_1 - p_2)}{\rho_1 g} = \frac{fL}{D}\frac{w_1^2}{2g}$$

or

$$w_1 = \sqrt{\frac{2D(p_1 - p_2)}{f(w_1)\rho_1 L}}. \tag{31.20}$$

The Darcy friction factor *f* is obtained from the Rose or Moody diagrams [11], [12], Figure 31.22. It requires an additional parameter: the relative pipe roughness

$$\frac{\varepsilon}{D}. \tag{31.21}$$

The nonlinear equation (31.20) is solved by direct iterations to find the flow velocity w_1. Finally, Eq. (31.19) gives the mass flow rate.

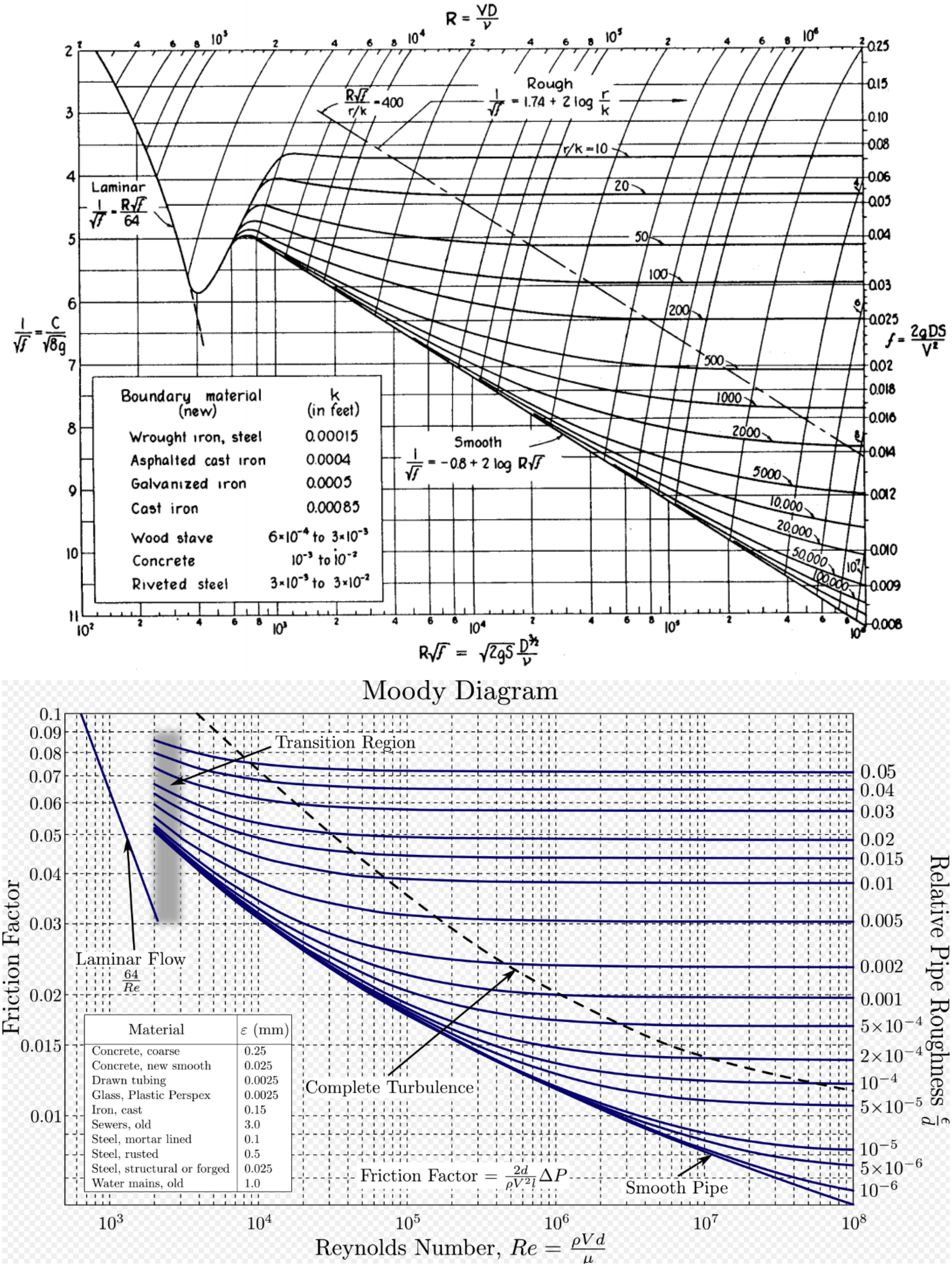
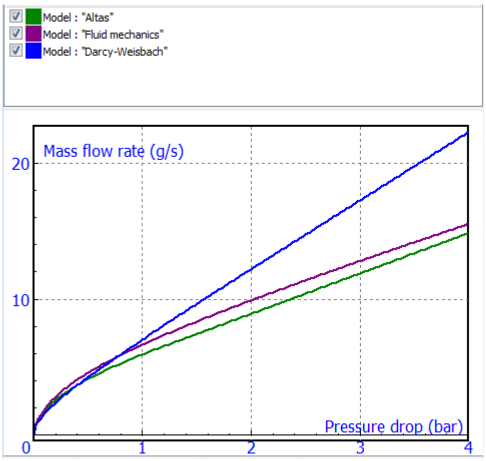


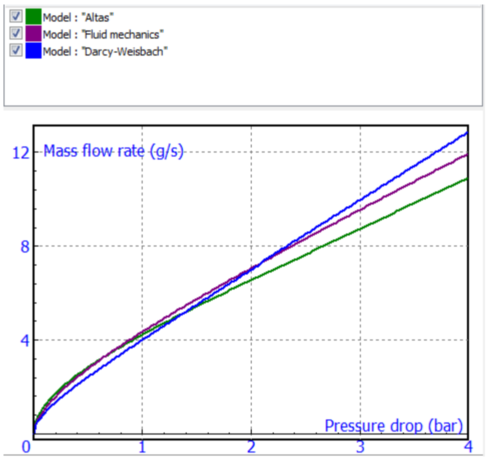
Figure 31.22. Rose and Moody diagram for friction factor

31.2.5.1.4. Comparison of models

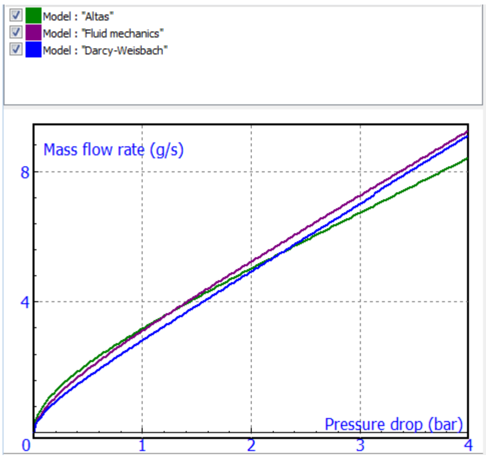
"Atlas" and "Fluid mechanics" mass flow rate equations give similar result for different length of lines, Figure 31.23. The Darcy-Weisbach equation can be used as well for long lines if the pipe roughness is $\varepsilon/D \sim 0.01$ for $p_2 \sim 0$ and $\varepsilon/D \sim 0$ for $p_2 \sim 5$ bar. The laminar flow region with a linear grows of the flow rate is shown in Figure 31.24 for a small pressure drop.



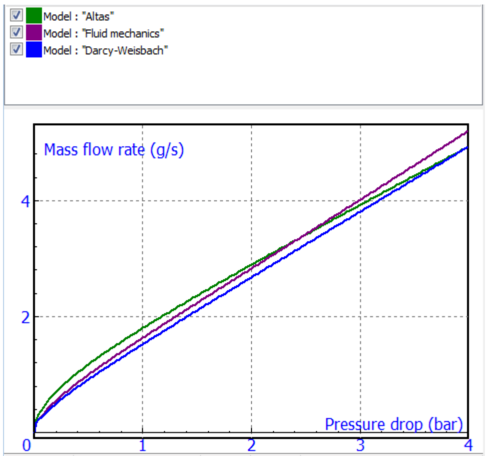
L=0.5m



L=1.5m



L=3m



L=10m

Figure 31.23. Comparison of mass flow rate models for different line length, D=5mm.

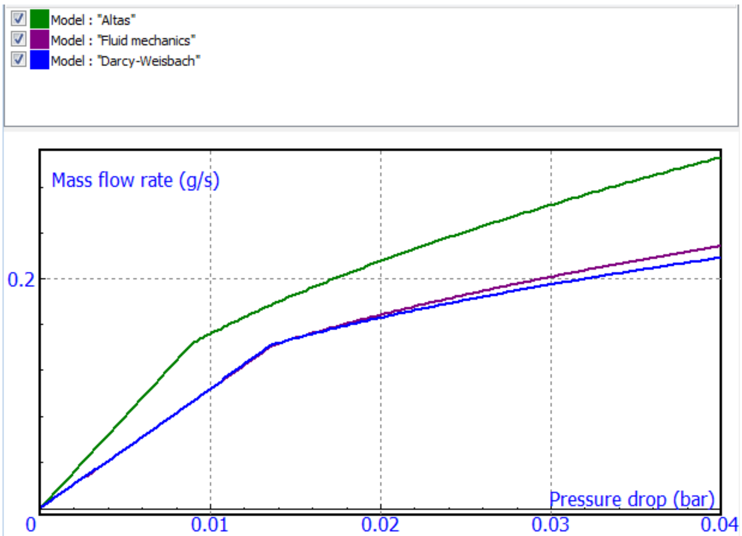


Figure 31.24. Comparison of mass flow rate models for small pressure drop, L=10m, D=5mm.

31.2.5.2. Dynamic pipeline model

It is known that the stationary model of a pipeline flow can be used for the low frequency processes only [8]. In contrary, the time domain model, which we consider here, can be applied both for low and high frequencies. The model cannot be used for description of supersonic processes and shock waves.

31.2.5.2.1. Mathematical model

The dynamic pipeline model implemented in UM is a slightly generalized version of the model described in [8]. Firstly, the model includes the continuity equation

$$\frac{\partial\rho}{\partial t}+\frac{\partial\rho w}{\partial x}=\frac{\partial\rho}{\partial t}+w\frac{\partial\rho}{\partial x}+\rho\frac{\partial w}{\partial x}=\frac{d\rho}{dt}+\rho\frac{\partial w}{\partial x}=0, \tag{31.22}$$

where w is the flow velocity, and ρ is the density.

It is assumed that the polytropic process take place for the air inside the pipeline,

$$p\rho^{-n}=c=\text{const}. \tag{31.23}$$

Differentiation of Eq. (31.23) with respect to t gives

$$\rho^{-n}\frac{dp}{dt}-np\rho^{-n-1}\frac{d\rho}{dt}=0$$

or

$$\frac{d\rho}{dt}=\frac{\rho}{np}\frac{dp}{dt} \tag{31.24}$$

Substituting Eq. (31.24) in Eq. (31.22) and assuming

$$\frac{\partial\dot m}{\partial x}=\frac{\partial A\rho w}{\partial x}\approx A\rho\frac{\partial w}{\partial x}$$

we obtain the first equation

$$\frac{dp}{dt}+\frac{np}{A\rho}\frac{\partial\dot m}{\partial x}=0 \tag{31.25}$$

In [8] an isothermal process for the long lines is assumed, which can be obtained from Eq. (31.25) for polytropic index n=1 and taking into account the ideal gas law $p=\rho RT$.

The second equation is derived in [8] and corresponds to the equation of motion of a short section

$$\frac{d\dot m}{dt}+\mathrm{A}\frac{\partial p}{\partial x}=-A\frac{\lambda\rho}{2D}w^2. \tag{31.26}$$

Here λ is the friction factor, see Sect. 31.2.5.1.2 *Mass flow rate model "Fluid mechanics"*, and D, A are the diameter and section area of the pipe.

Following [8], consider an algorithm for numerical solving Eq. (31.25), (31.26). The line is divided into N segments of the equal length $\Delta L=L/N$. The continues variables $p=p(x,t),\dot m=\dot m(x,t)$ are replaced by the discrete variables at the segment ends

$$p_i=p(x_i,t),\quad \dot m_i=\dot m(x_i,t),$$
$$x_i=\Delta Li,\qquad i=0,\dots,N.$$

The partial derivatives with respect to the coordinate x are replaces by the left or right finite differences (the right differences are written below)

$$\left.\frac{\partial p}{\partial x}\right|_{x=x_i}=\frac{p_{i+1}-p_i}{\Delta L},\qquad \left.\frac{\partial\dot m}{\partial x}\right|_{x=x_i}=\frac{\dot m_{i+1}-\dot m_i}{\Delta L}$$

and the following system of ordinary differential equations are solved by UM

$$\frac{dp_i}{dt} = -\frac{p_{i+1} - p_i}{\Delta L} - \frac{np_i}{A\rho_i \Delta L}(\dot{m}_{i+1} - \dot{m}_i) \tag{31.27}$$

$$\frac{d\dot{m}_i}{dt} = -\frac{\dot{m}_{i+1} - \dot{m}_i}{\Delta L} - A\frac{p_{i+1} - p_i}{\Delta L} - A\frac{\lambda_i \rho_i}{2D} {w_i}^2$$

Equations (31.27) are written with the right finite differences. If necessary, they are replaced by the left ones, e.g. for $i = N$. The number of equations depends on boundary conditions. For example, if the pipeline connects two chambers, the pressure at begin and end of the pipeline are equal to the chamber pressures, and the pressure equations for $i = 0$ and $i = N$ are omitted. The total number of equations in this case is $2N$.

31.2.5.2.2. Verification of time domain model

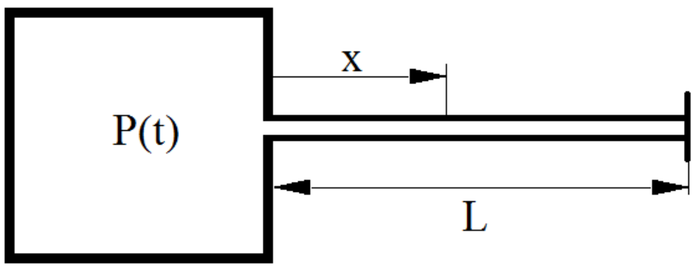


Figure 31.25. Sketch of experimental equipment for verification of time domain model of pipeline

Following to the book [8], consider simulation of the flow dynamics in a long pipe according to the experimental equipment in Figure 31.25. The inlet pressure of the pipe is a given function of time $p(0) = P(t)$, while the opposite end is locked. Consider comparison of simulation and experimental results for $p(L)$ when either step or harmonic functions are considered as $P(t)$. For the test we use the UM tool described in Sect. 31.3.2.3.3 *Player for time domain pipeline model*.

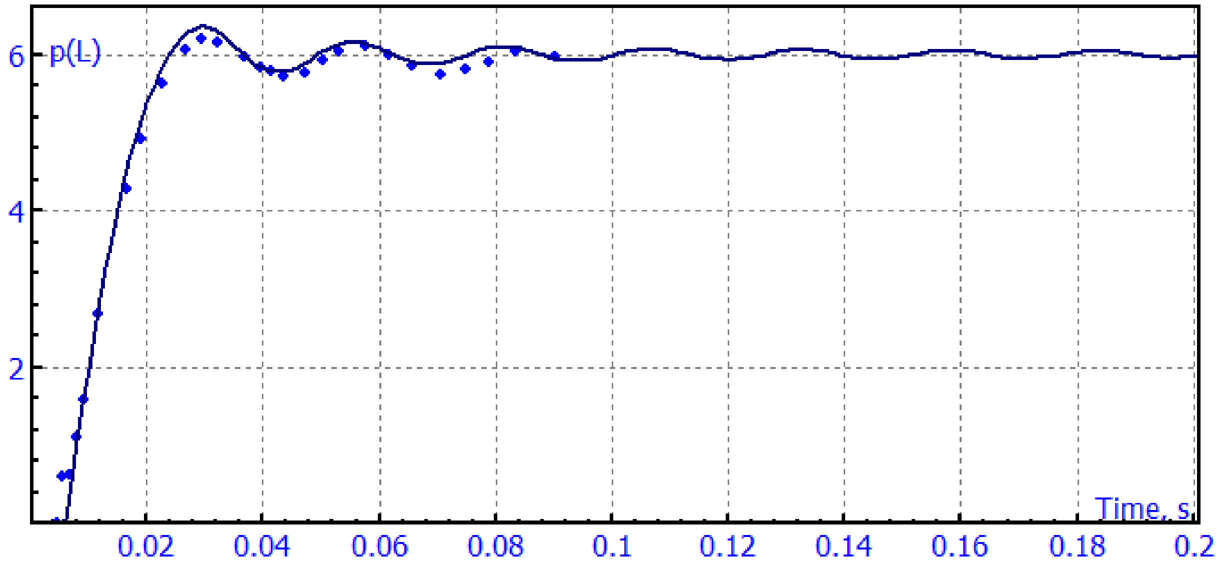


Figure 31.26. Step response. Comparison with experiment

Firstly, consider the step response, where the step function is approximated by the following expression:

$$P(t) = P_0\left(1 - e^{-t/T_s}\right)$$

with $P_0 = 6$ bar and $T_s = 200$ s. Comparison of simulation with experimental results from [8] is shown in Figure 31.26. The value of the polytropic index in this test is *n* = 1.08.

The next two tests are related to the response to harmonic excitations. In simulation we use the gliding frequency excitation

$$P(t) = P_0 + \Delta P \sin(2\pi(f_0 + \epsilon t/2)t).$$

In particular, the test allows us to compare the natural frequencies of the air in the pipeline with experimental and theoretical values. The theoretical natural frequencies of air in a pipeline in Figure 31.25 are

$$f_i = \frac{ic}{4L}, i = 1,3,5\ ...$$

Here *c* is the sound velocity

$$c = \sqrt{\gamma \frac{p}{\rho}}$$

with γ=1.4.

Following the book [8] we consider two tests:

1. L = 1m, D = 2.5mm, $P_0 = 7$bar, $\Delta P = 0.1$bar, N = 15
2. L = 25m, D = 5.5mm, $P_0 = 7$bar, $\Delta P = 0.1$bar, N = 35

The first natural theoretical frequencies f_1 are 85.8Hz for Case 1 and 3.43Hz for Case 2.

Comparison of simulation and experimental results for the gain factor $(P(L) - P_0)/\Delta P$ is given in figures below. Experimental data from [8] are plotted by the marker.

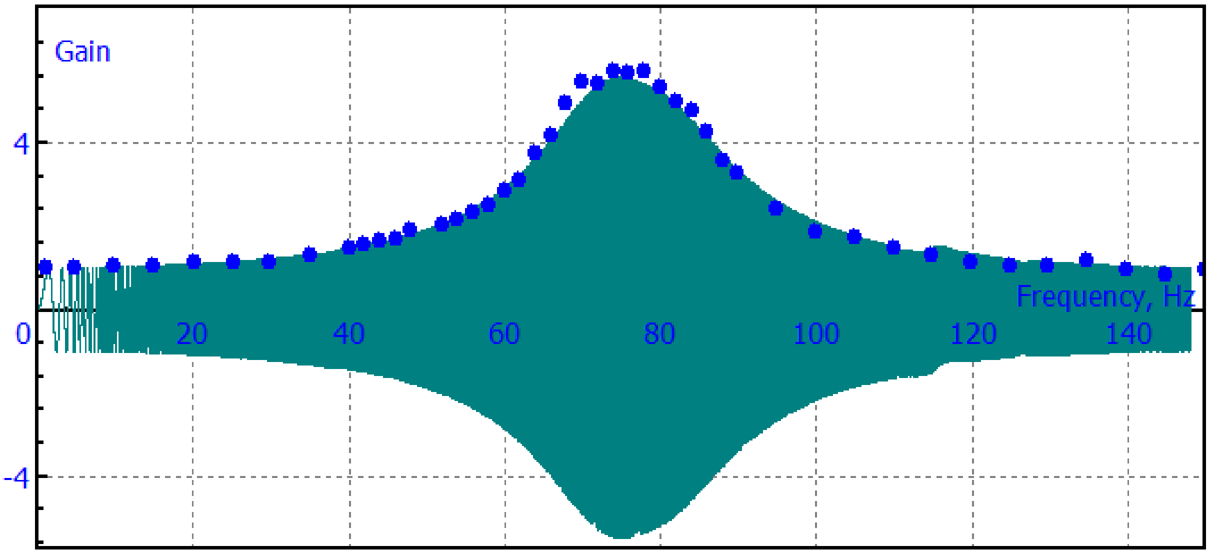


Figure 31.27. Frequency response for Case 1, the value of polytropic index *n* = 1.2

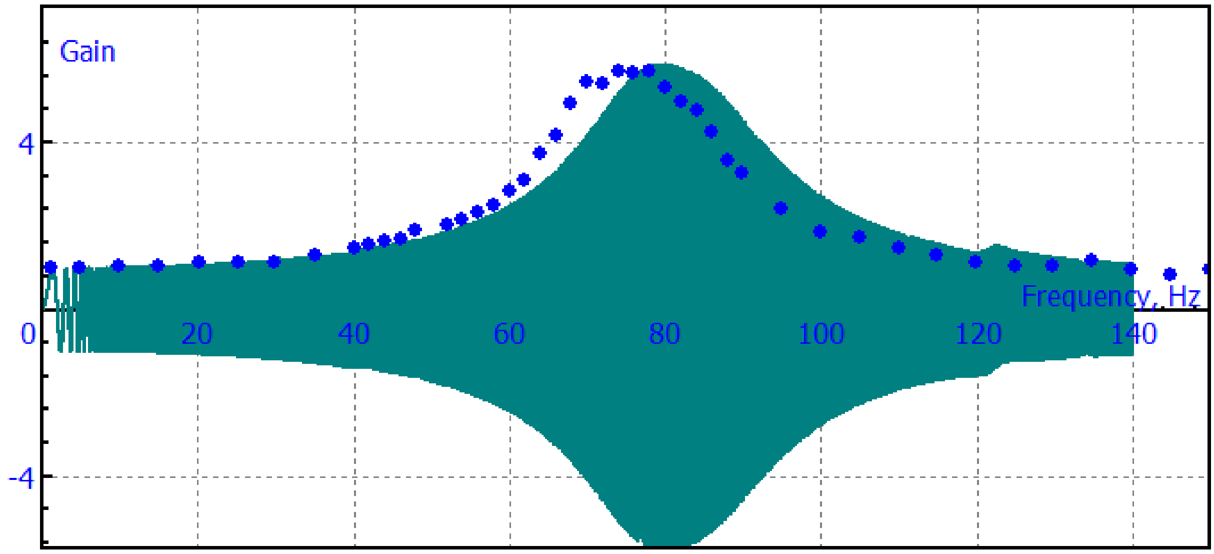


Figure 31.28. Frequency response for Case 1, the value of polytropic index n = 1.4

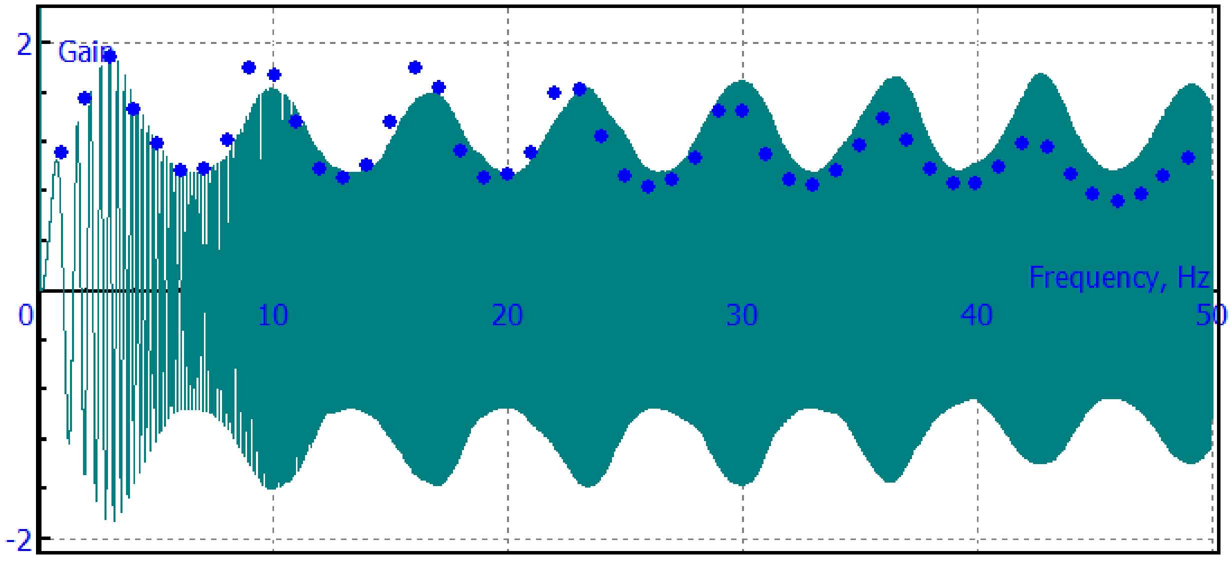


Figure 31.29. Frequency response for Case 2, the value of polytropic index n = 1.4

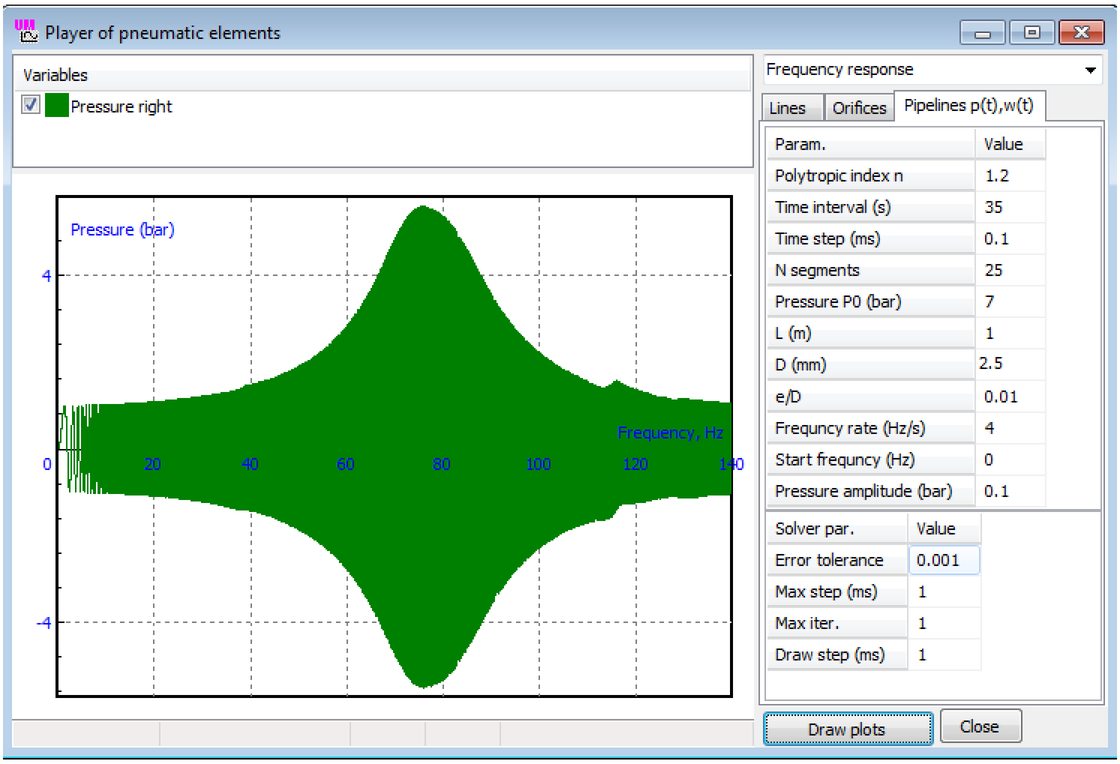


Figure 31.30. Simulation options for Case 1

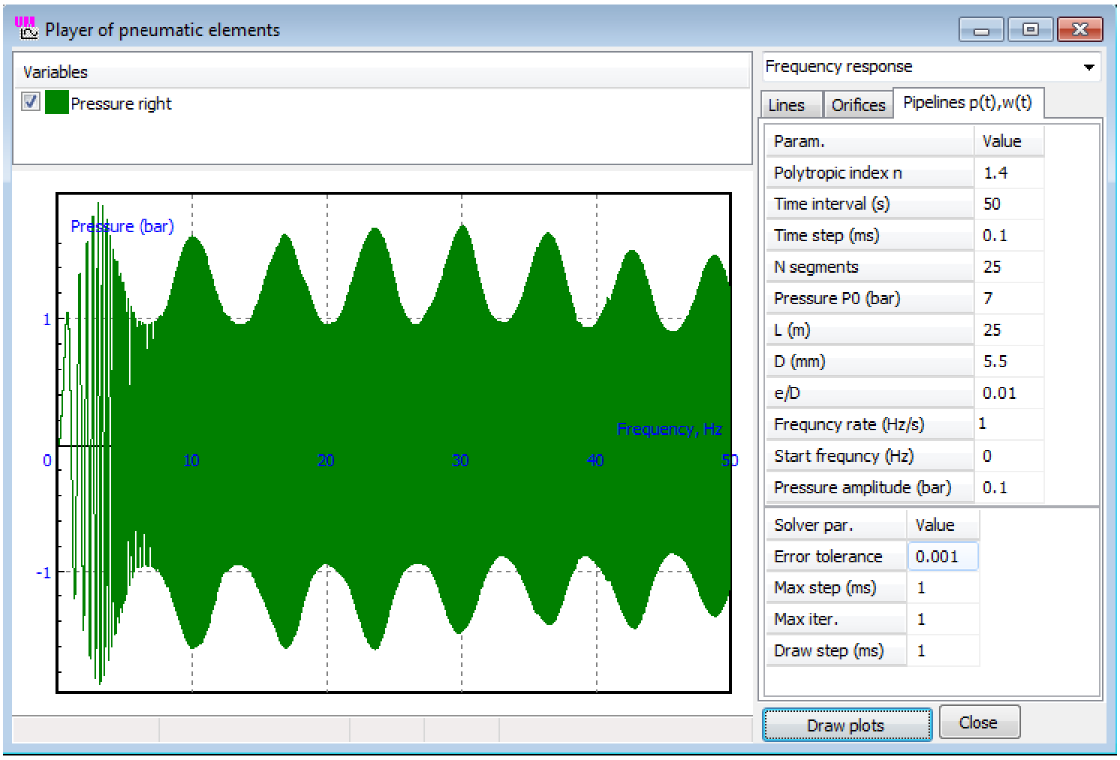


Figure 31.31. Simulation options for Case 2

31.2.6. Orifices

Similar to a line, an orifice (nozzle, valve) is a connection between two nodes of pneumatic system. The mathematical model of the orifice includes a dependence of the mass flow rate on the pressure drop.

31.2.6.1. Nozzle

Let $p_1 > p_2$ be the pressures in the connected node. Then the mass flow rate for a nozzle is computed as [8]

$$\dot{m} = AC_d p_1 \sqrt{\frac{2}{RT_1}}\psi(r_p), \tag{31.28}$$

$$\psi(r_p) = \begin{cases} \sqrt{\frac{\gamma}{\gamma-1}\left(r_p^{2/\gamma} - r_p^{(\gamma+1)/\gamma}\right)}, r_p > b \\ \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma+1}}\sqrt{\frac{\gamma}{\gamma+1}} = 0.484, r_p \le b \end{cases}$$

$$b = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 0.528,$$

$$r_p = \frac{p_2}{p_1}.$$

Here A is the nozzle area, C_d is the discharge coefficient. The parameters are given in SI (Pa, m^2, m^3).

The discharge coefficient $C_d \le 1$ depends on the nozzle geometry. In particular, for the well rounded nozzle $C_d = 1$. Values of this parameter for different sharp nuzzles can be found in [8].

Laminar flow.

Similar to the ISO 6358 model, the derivative of the mass flow rate tends to infinity when the pressure drop goes to zero (31.12). Consider Eq. (31.28) for small pressure drop

$$r_p = 1 - \epsilon, \epsilon \ll 1$$

and derive the pressure ratio for the boundary Remolds number Re^*=2000.

$$\psi(r_p) = \sqrt{\frac{\gamma}{\gamma-1}\left((1-\epsilon)^{2/\gamma} - (1-\epsilon)^{(\gamma+1)/\gamma}\right)} \approx \sqrt{\frac{\gamma}{\gamma-1}(-\epsilon\, 2/\gamma + \epsilon\,(\gamma+1)/\gamma)} = \sqrt{\epsilon},$$

$$\mathrm{Re}^* = \frac{\dot{m}^* D}{A\mu} = \frac{AC_d p_1 D}{A\mu}\sqrt{\frac{2}{RT_1}}\sqrt{\epsilon^*},$$

$$r_p^* = 1 - \epsilon^* = 1 - \frac{RT_1}{2}\left(\frac{A\mu \mathrm{Re}^*}{AC_d p_1 D}\right)^2.$$

The final expression for the laminar region is

$$\dot{m} = \dot{m}^* \frac{1-r_b}{1-r_b^*}, \quad r_b = \frac{p_2}{p_1} > r_b^*, \tag{31.29}$$

$$\dot{m}^* = AC_d p_1 \sqrt{\frac{2\epsilon^*}{RT_1}}.$$

Example for a circular orifice D = 5mm: ϵ^* ≈0.00022, $r_b^* \approx 0.99978$.

31.2.6.2. ISO 6358

The ISO 6358 model for the flow rate [8] is already used above in Sect. 31.2.5.1.1 *Mass flow rate model "Atlas"*, Eq. (31.10). Here we point out that for the "Orifice" element we use this model in two forms:

- **Standard**. The user should set the numerical value of the sonic conductance *C*, and the critical pressure ratio *b*.
- The ratio C/d^2 model for an restriction offered in [13], [14]:
  $\frac{C}{d^2}$ = 8 dm^3/(min·bar·mm^2) = 1.33×10^{-9} m^3/(s·Pa·mm^2)

31.2.6.3. Comparison of orifice models

The nozzle model for definite values of the discharge coefficient C_d and the pressure ratio b give results very close to those from Sect. 31.2.6.2 ISO 6358. To obtain the correspondence of the parameters C_d and *C*, consider condition of the equal choked flow at $p_2/p_1 = b$ for Eqs. (31.10) and (31.28)

$$p_1 C \rho_0 \sqrt{\frac{T_0}{T_1}} = AC_d p_1 \sqrt{\frac{2}{RT_1}} \psi(b) = \frac{1}{4}\pi d^2 10^{-6} C_d p_1 \sqrt{\frac{2}{RT_1}} 4.84,$$

$$\frac{C}{d^2} = 0.121 \cdot 10^{-6} \frac{\pi}{\rho_0} \sqrt{\frac{2}{RT_0}} = 1.539 \cdot 10^{-9} C_d$$

The orifice diameter in this formula is measured in millimeters.

Thus, the nozzle model gives similar results with the ISO 6358 ratio C/d^2 model from the previous section for the following value of the discharge coefficient, Figure 31.32:

$$C_d = 0.866.$$

Models are compared for small pressure drop in Figure 31.33 to illustrate the region of the laminar flow.

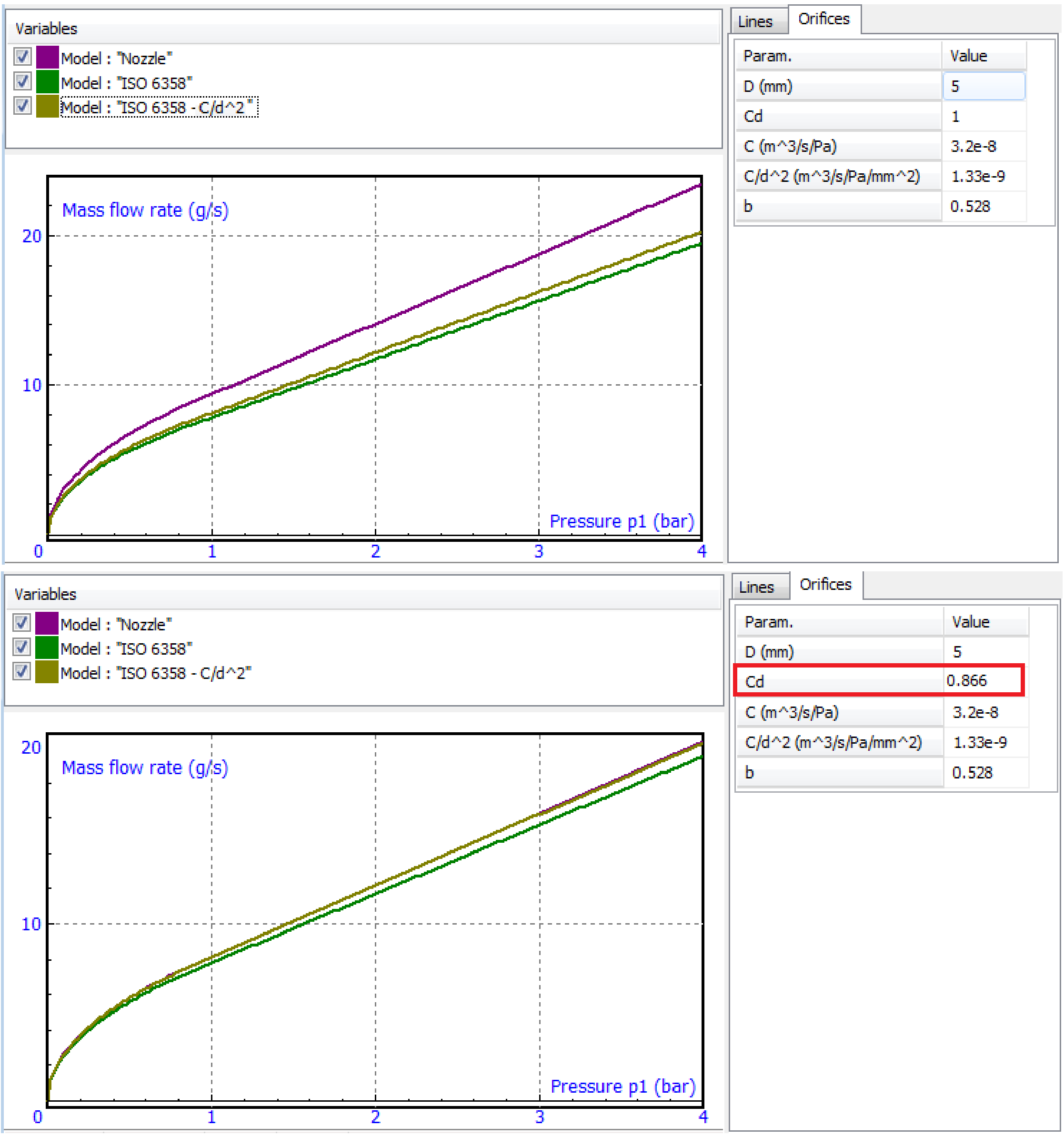


Figure 31.32. Comparison of mass flow rate for different orifice models

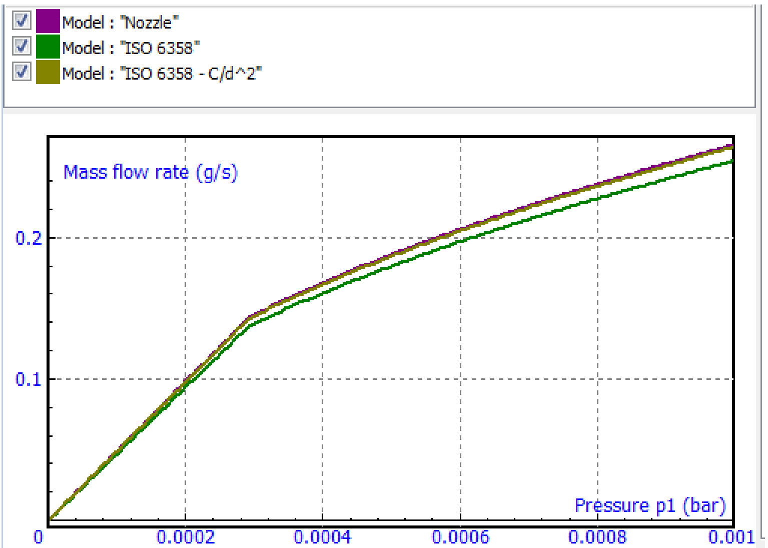


Figure 31.33. Comparison orifice models for small pressure drop

31.3. Pneumatic systems

A pneumatic system (PS) in UM is considered as a graph, which nodes are connected by edges. Each node of a graph corresponds to one of the pneumatic elements

- Rigid chamber, Sect. 31.2.2;
- Air spring, Sect. 31.2.3;
- Simple node, Sect. 31.2.4.

The graph edges are

- Pneumatic lines, Sect. 31.2.5;
- Orifices, Sect. 31.2.6.

31.3.1. Parameters of tabular air springs

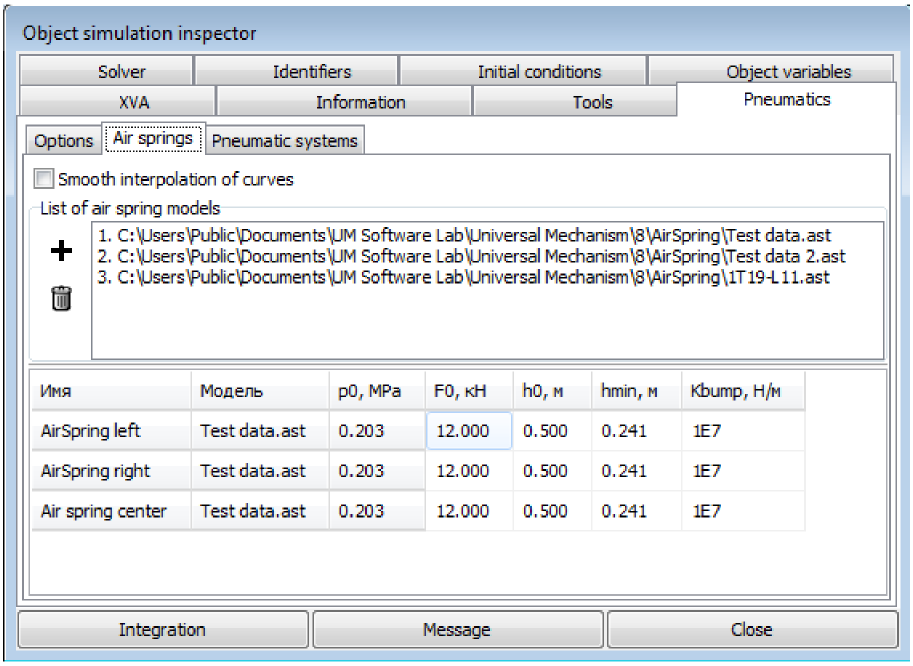


Figure 31.34. Tabular air springs parameters

Tabular air spring (AS) parameters are entered on the **Pneumatics | Air springs** tab, Figure 31.34.

List of tabular AS force element is shown in the tab bottom. All AS should be added to the model as special force elements in UM Input program, Sect. 31.2.3.1.1 *Parameters of tabular air spring in Input program*.

Files *.ast contain tabular AS models, Sect. 31.2.3.1.3.4 *Creating UM files *.ast with tabular air spring models*. By default, the files are located in the directory **{UM data}\AirSpring.**

List of tabular AS models must include at least one element. Use the **+** and buttons to add and remove models from the list.

Double click on the file name in the list of AS models to open the window with the corresponding model, Figure 31.9.

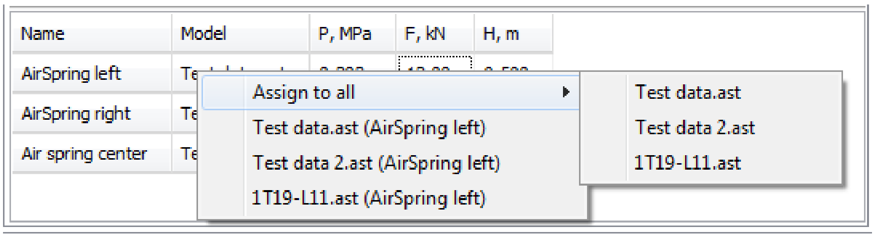


Figure 31.35. Assignment of AS model

To assign a model to AS force element, move the mouse cursor to the necessary force in the list, click the right mouse button and select the tabular model in the popup menu, Figure 31.35.

Name	Model	p0, MPa	F0, kN	h0, m	hmin, m	Kbump, N/m
AirSpring left	Test data.ast	0.203	12.000	0.500	0.241	1E7
AirSpring right	Test data.ast	0.203	12.000	0.500	0.241	1E7
Air spring center	Test data.ast	0.203	12.000	0.500	0.241	1E7

Figure 31.36. Air spring force elements with assigned tabular model and static values of force and height

Set the static force and height F_0, h_0 to each of the force elements, Figure 31.36. The corresponding pressure p_0 is computed automatically.

The minimal height (**hmin**) and the **bump stop stiffness constant** (**Kbump**) for each of the AS are important if the tabular model does not include the bump stop description. Nonlinear bump stop models can be described by additional force elements in UM Input program in parallel to the AS force elements; in such cases, zero values should be set for the **Kbump** parameter.

Double click on the force element with assigned tabular model and static parameters to get plots for force, volume and pressure versus height according to the **dynamic load model,** Figure 31.37**, Sect. 31.2.3.1.4** *Mathematical model of air spring by tabular description***.**

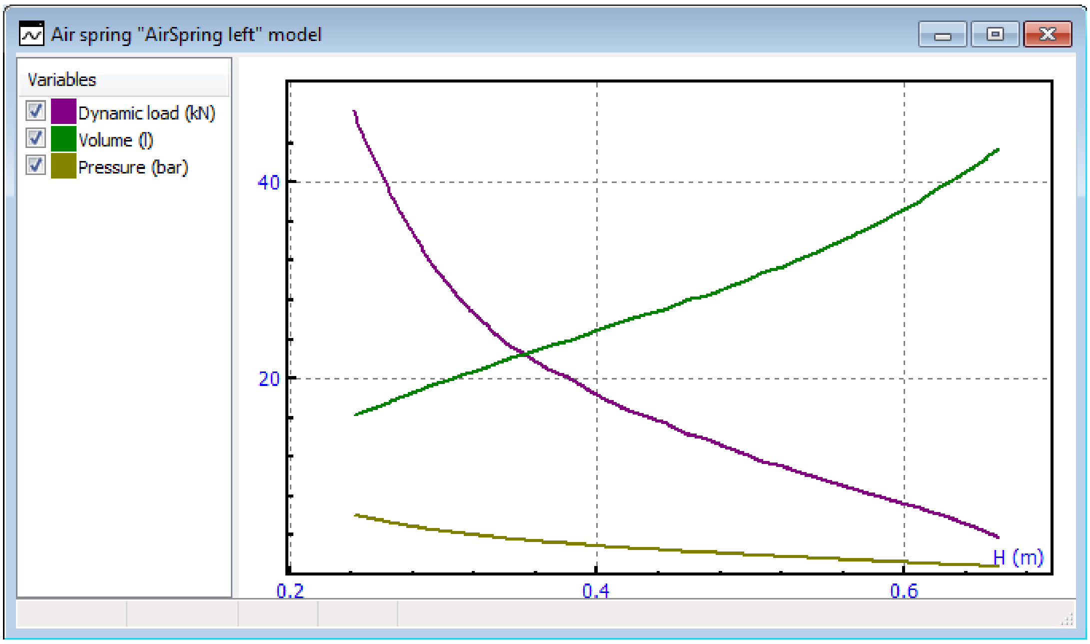


Figure 31.37. Dynamic load model of air spring

Check the **Smooth interpolation of curves** option

- to interpolate the static tabular data for force and volume versus height by B-splines;
- to interpolate the static force a volume data versus pressure by polynomials of second order.

If an *AS is isolated* and not connected to other elements of a PS, its air mass is constant. In this case the AS simulation properties are completely described by the dynamic load model, Figure 31.37. If the AS is connected by pneumatic lines and orifices with other nodes of a PS, its behavior is defined according to the solution a system of nonlinear equations of PS at each step of the simulation process.

31.3.2. Description of pneumatic systems

A PS is described by a graph, which vertices or nodes are simple nodes, air springs and chambers. PS graph edges are pneumatic lines and orifices connecting the nodes.

31.3.2.1. List of pneumatic systems

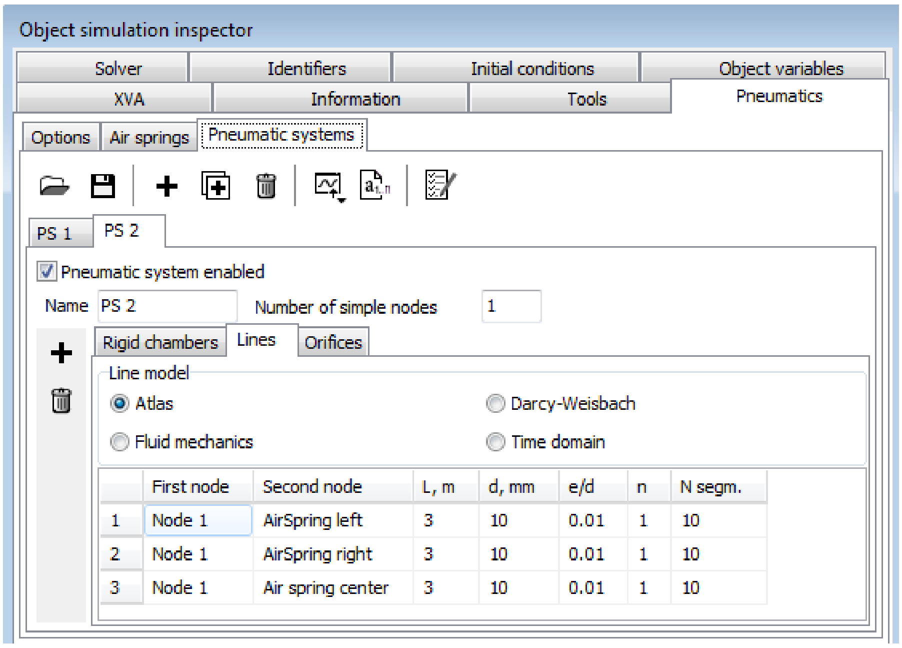


Figure 31.38. List of pneumatic systems

UM Model can include several PS, which are not interacting pneumatically. Buttons for managing the PS list as well as for call of some useful tools are located in the top of the **Pneumatic systems** tab, Figure 31.38.

- read *.psc file with description of PS list.

- save description of PS list in a *.psc file.

The description of the PS list is saved automatically in the general configuration file *.icf, in particular, in the last.isf file. The user can save the same description in an extra file *.psc, which could be useful in some cases.

- add a new PS to the list.

- add a copy of the current PS to the list.

- delete the current PS.

- open a player of line and orifice models, see Sect. 31.3.2.3.

- open a list of pneumatic variables; see Sect 31.3.2.4 *List of variables for pneumatic elements*.

- verify description of PS, Sect. 31.5.1 *Errors in pneumatic system model*.

Pneumatic systems can be either active (enabled) or not active (disabled). To disable a PS, uncheck the **Pneumatic system enabled** option, Figure 31.39.

Remark	If the user described several PS, he must remember that the enabled pneumatic systems must be *completely pneumatically independent on each other*. As a consequence, a tabular AS can be connected with one enabled PS only.

31.3.2.2. Description of pneumatic system

31.3.2.2.1. General parameters of pneumatic system

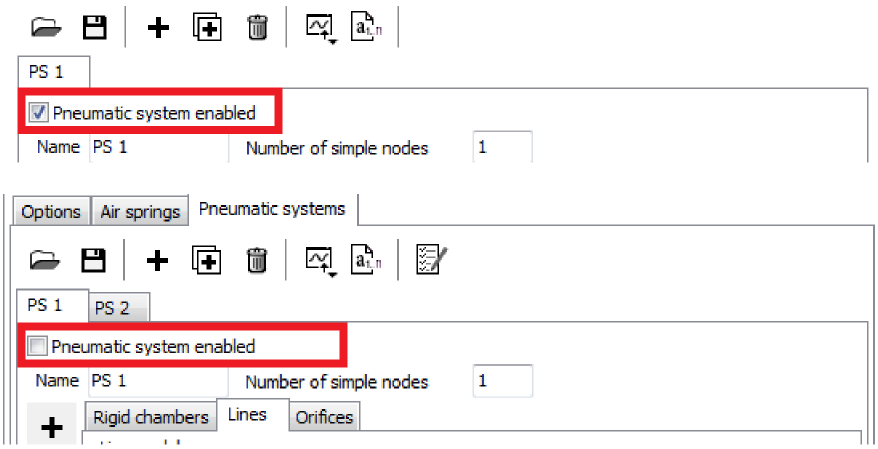


Figure 31.39. Enabled and disabled PS

Any PS in the list can be either **enabled or disabled**. The current PS is enabled, if the corresponding option is checked, Figure 31.39. Enabling and disabling allow the user a simple comparison of different variants of PS.

Remark	If a tabular AS is included in disabled PS only or not connected to any PS, the AS is considered as an isolated one and its simulation properties are computed according to the dynamic load model, Figure 31.37.

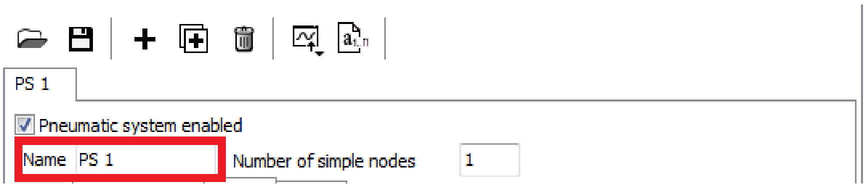


Figure 31.40. Name of PS

The **name of PS** identifies the PS in the case of several PS in the list, Figure 31.40.

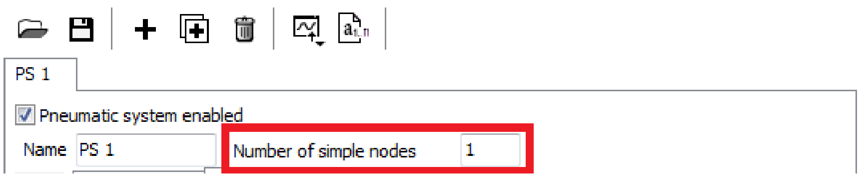


Figure 31.41. Number of simple nodes

Set the **number of simple nodes** if such elements are presented in the current PS, Figure 31.41, Sect. 31.2.4 *Simple nodes*.

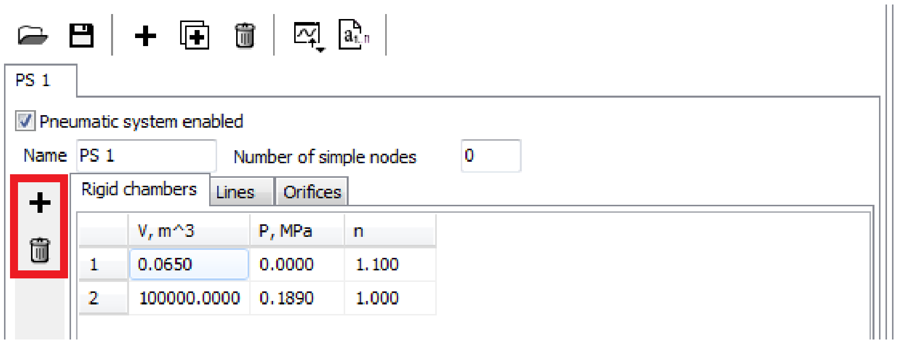


Figure 31.42. Buttons for adding and deleting pneumatic elements

Lists of pneumatic elements except AS force elements are created dynamically as parts of PS. The standard buttons are used for adding and deleting elements of lists of chambers, lines and orifices, Figure 31.42.

31.3.2.2.2. List of rigid chambers

Description of rigid chambers includes the following numerical parameters, Figure 31.42:

- Volume V, m^3
- Initial pressure P, MPa
- Polytropic index, n.

Remark To describe a connection to a source with a constant pressure, e.g. to environment, a rigid chamber with a large volume is used, see the second line in Figure 31.42.

31.3.2.2.3. List of pneumatic lines

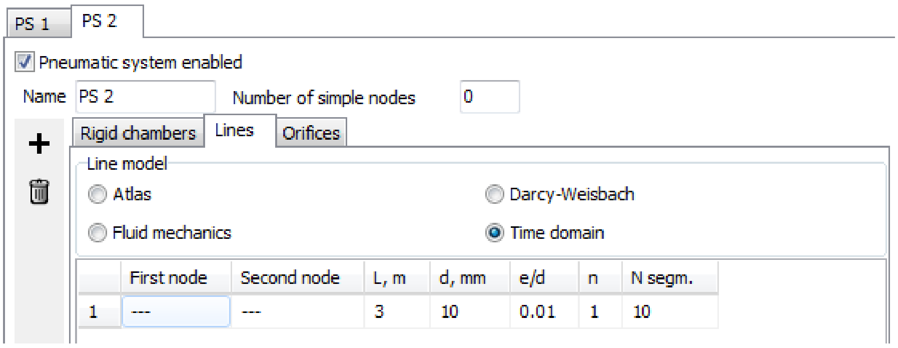


Figure 31.43. New pneumatic line

The user should select the **model of pneumatic lines**, Figure 31.43, see Sect. 31.2.5 *Pneumatic lines*.

The following **numeric parameters** should be set in the parameter table:

- Length of line L, m
- Line diameter d, mm
- The relative pipe roughness e/d (is used for the Darcy-Weisbach and Time domain models, Sect. 31.2.5.1.3 *Darcy-Weisbach equation*, 31.2.5.2 *Dynamic pipeline model*)
- The polytropic index **n** and the number of segments **N** are used for the Time domain model only, Sect. *31.2.5.2 Dynamic pipeline model.*

To assign **nodes connected by a line**, double click by the mouse on the cell and select a node from the list, Figure 31.44. The node can be changed in the same manner.

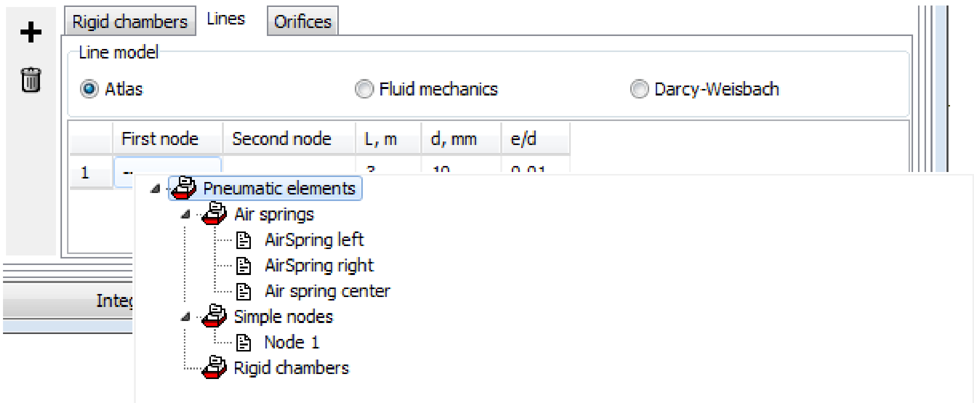


Figure 31.44. Assignment of nodes to a line

Check the **Nikuradse–Prandtl–Karman (NPK) equation for smooth pipe** option to use the corresponding model for the friction factor (31.17) otherwise the Blasius equation (31.16) is used. This factor is used for the **Fluid mechanics** model only.

31.3.2.2.4. List of orifices

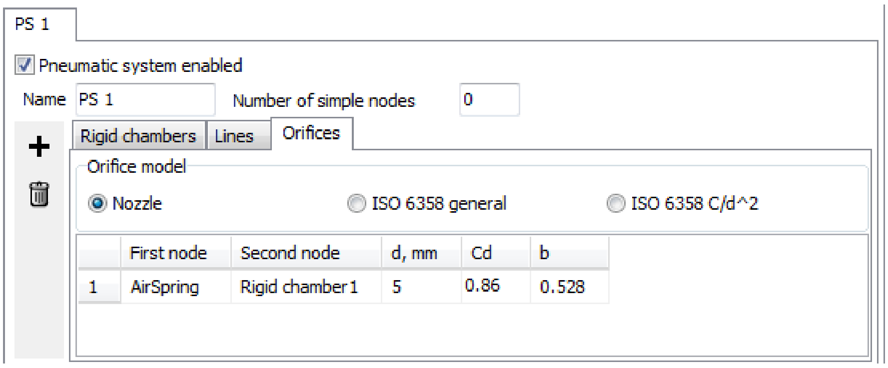


Figure 31.45. List of orifices

The user should select the **Orifice model**, Figure 31.45, see Sect. 31.2.6.

The following **numeric parameters** should be set in the parameter table:

- The orifice diameter d, mm
- Depending on the orifice model
  - the discharge coefficient Cd (nozzle),
  - the sonic conductance C (ISO 6358 general),
  - the ratio C/d^2 (ISO 6358 C/d^2)
- The critical pressure ratio *b*.

Nodes connected by the orifice are assigned by the double click of the mouse on the cells , Figure 31.44.

31.3.2.3. Player for line and orifice models

Click on the button  (Figure 31.38) to open the window for graphical comparison of different mass flow rate stationary models for pipelines and orifices as well as for testing the time domain model of pipelines, Figure 31.47. The user can change the parameters in the right table and redraw the plots by the **Draw plots** button.

31.3.2.3.1. Stationary models of lines

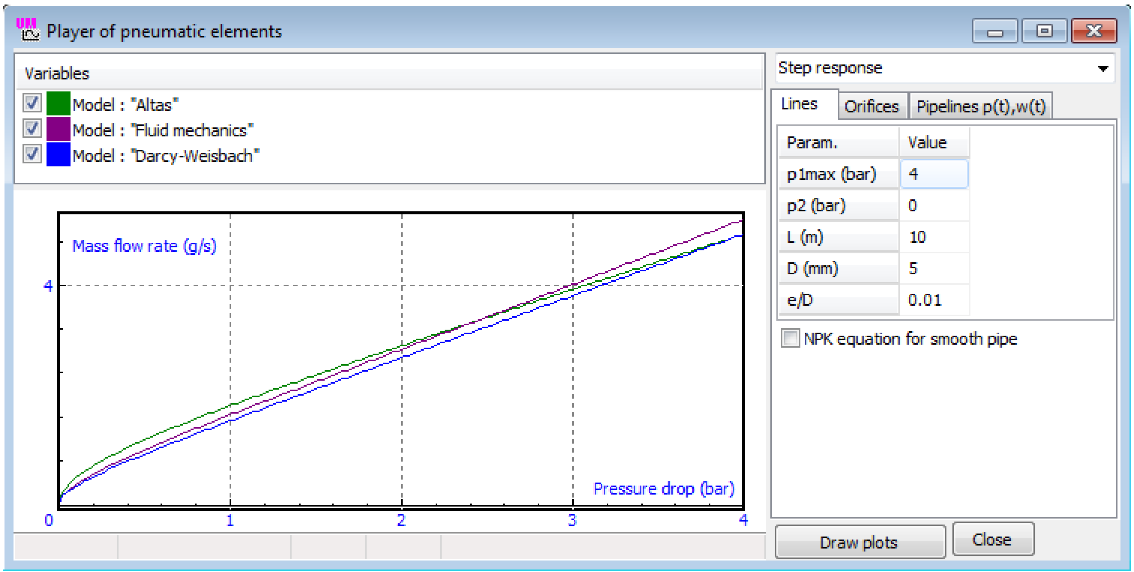


Figure 31.46. Comparison of stationary pipeline models

The gauge pressures are set in the table and drawn in bar. The mass flow rate verses the pressure drop $\Delta p = p_1 - p_2$ is computed for the given value pressure p_2 and different values of $p_1 > p_2$.

The relative pipe roughness **e/D** is used for the Darcy-Weisbach model, Sect. 31.2.5.1.3.

The **NPK equation for smooth pipe** option is applied to the "Fluid mechanics" model, Sect. 31.2.5.1.2.

31.3.2.3.2. Models of orifices

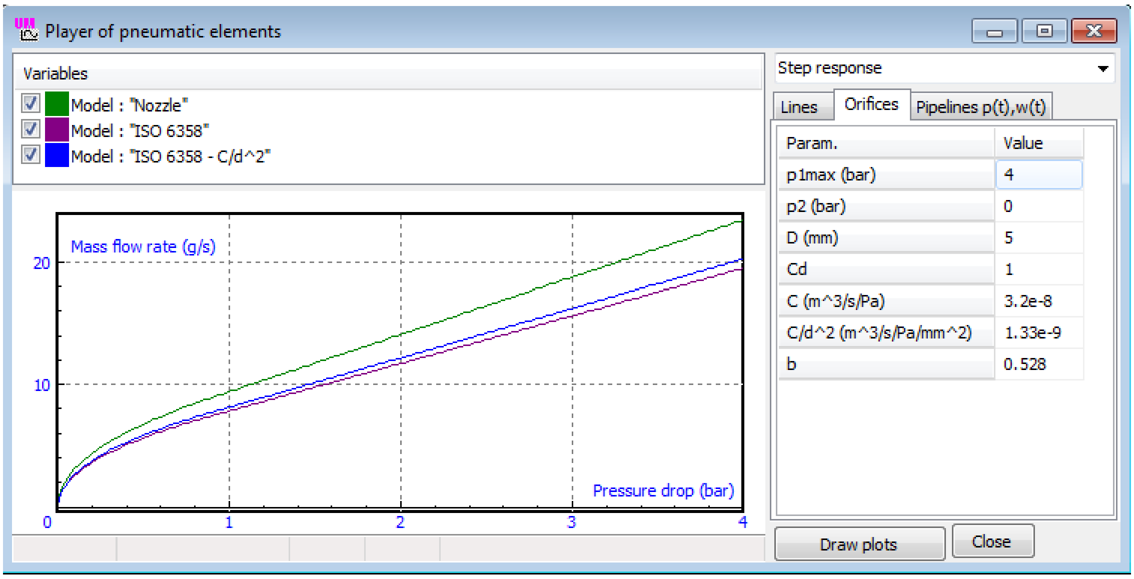


Figure 31.47. Comparison of orifice models

The gauge pressures are set in the table and drawn in bar. The mass flow rate verses the pressure drop $\Delta p = p_1 - p_2$ is computed for the given value pressure p_2 and different values of $p_1 > p_2$.

Cd is the discharge coefficient in the "Nozzle" orifice model, Sect. 31.2.6.1.

C is the **standard** sonic conductance in the "ISO 6358" orifice model, Sect. 31.2.6.1.

C/d^2 is the ratio C/d^2 model for an restriction offered in [13], [14].

31.3.2.3.3. Player for time domain pipeline model

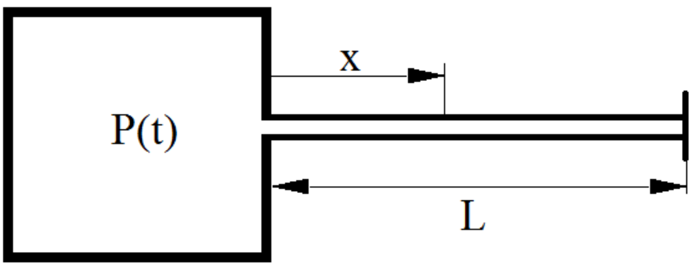


Figure 31.48. Model used in the player of dynamic pipeline

We implement the model of the pipeline excitation shown in Figure 31.48. The inlet pressure is a function of time $p(0) = P(t)$, and the pressure in the opposite locked end $p(L)$ is computed according to the time domain model described in Sect. 31.2.5.2.1.

The user can select the type of excitation from the drop down menu.

1. **Step response** $P(t) = P_0\left(1 - e^{-t/T_s}\right)$, Figure 31.49. T_s is the time constant specifying the smooth approximation of the step function. Smaller values of this constant increase the rate of the inlet pressure growth; see the **Pressure left** variable in Figure 31.49. The **Pressure right** variable is the pressure at the locked end of the line.

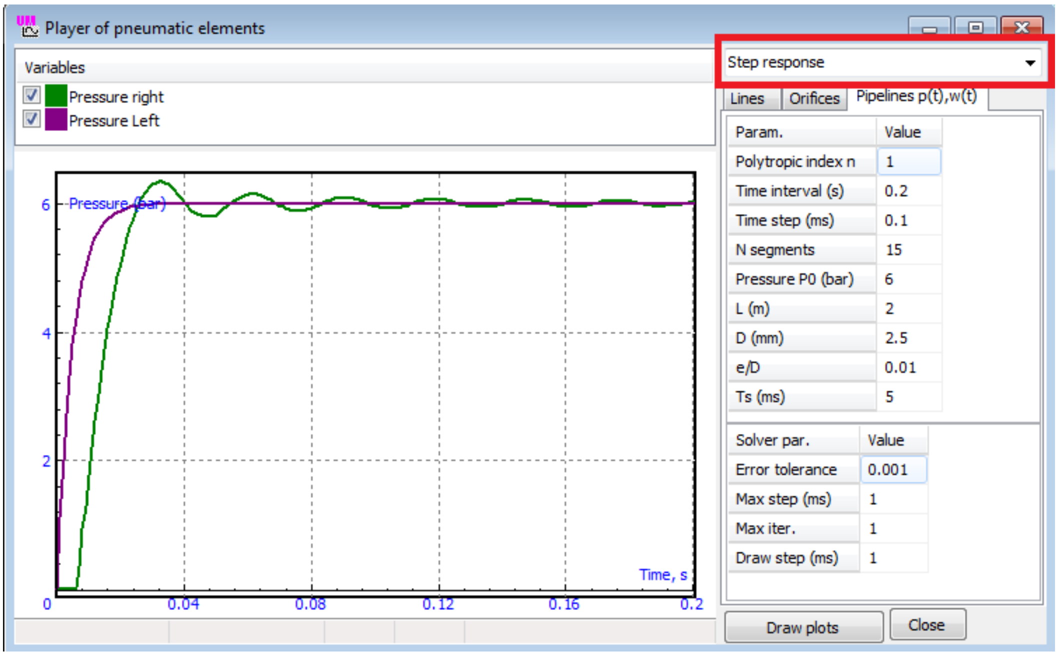


Figure 31.49. Step response

2. Frequency response $P(t) = P_0 + \Delta P \sin(2\pi(f_0 + \epsilon t/2)t)$. This model corresponds to the gliding frequency excitation, where the frequency decreases with the rate ϵ, $f = f_0 + \epsilon t$ Hz. The parameter ΔP corresponds to the **Pressure amplitude** row in the table, Figure 31.50.

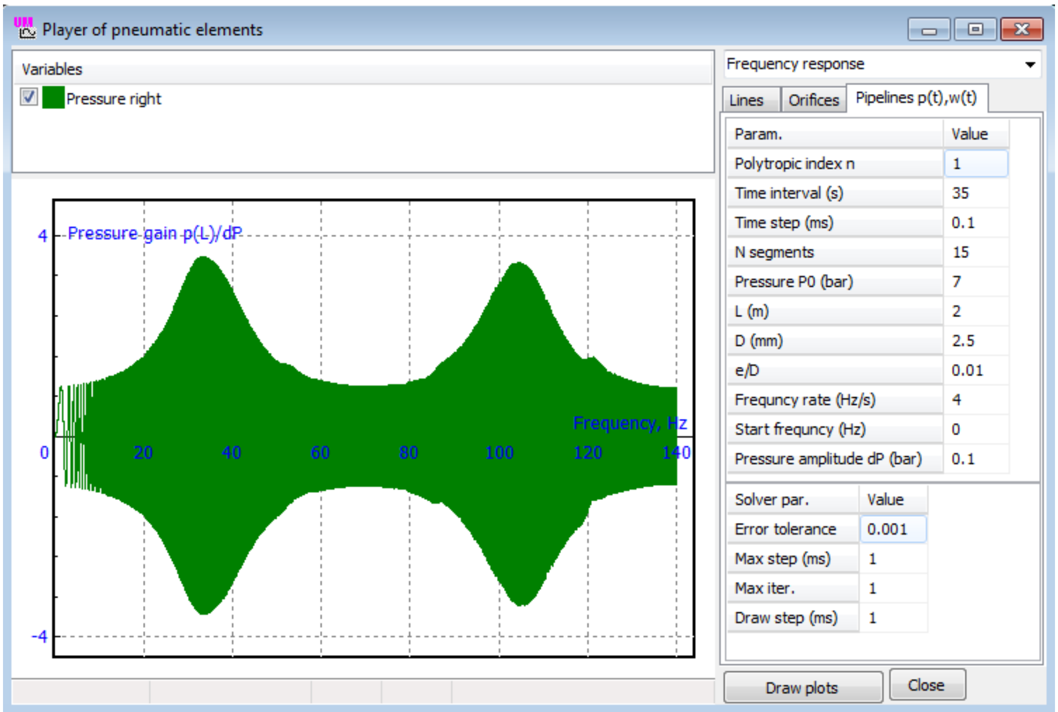


Figure 31.50. Frequency response

The user can get a single frequency response like in Figure 31.51 by setting zero value of the frequency rate, and the desired frequency value as the start frequency f_0.

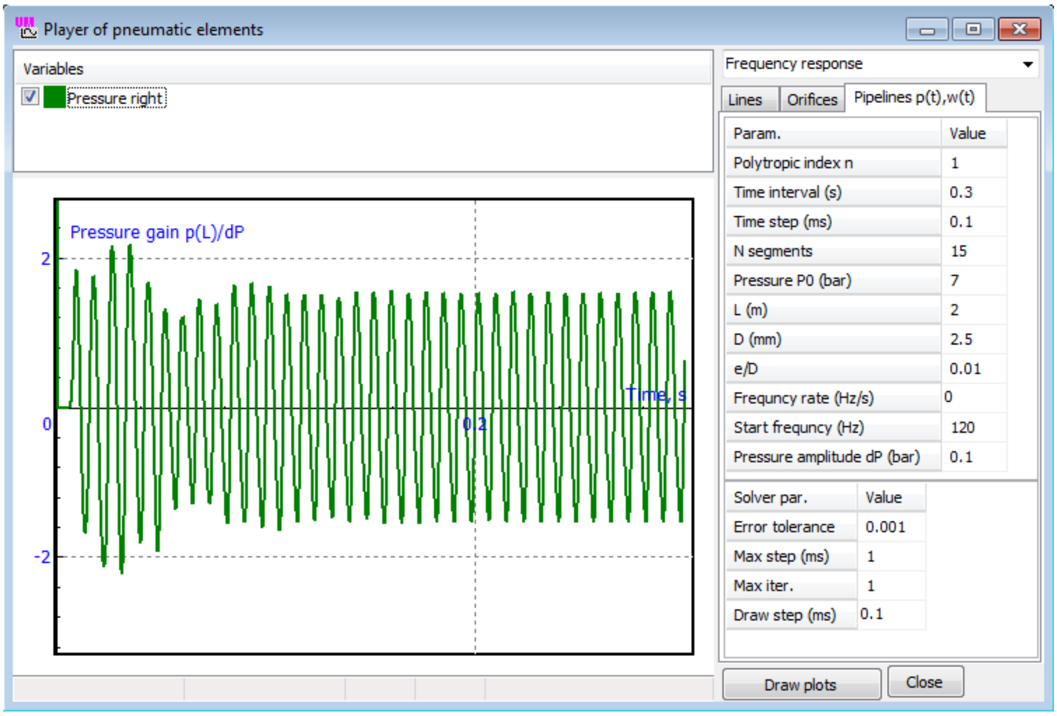


Figure 31.51. Single frequency response

31.3.2.4. List of variables for pneumatic elements

Click on the button (Figure 31.38) to generate the full list of pneumatic variables, Figure 31.52. The user can

- select a system of units: SI or Imperial, Table 2;
- add new pages with variables generated with the Master of Variables;
- save the list to a file;
- drag variables from the list to graphical windows.

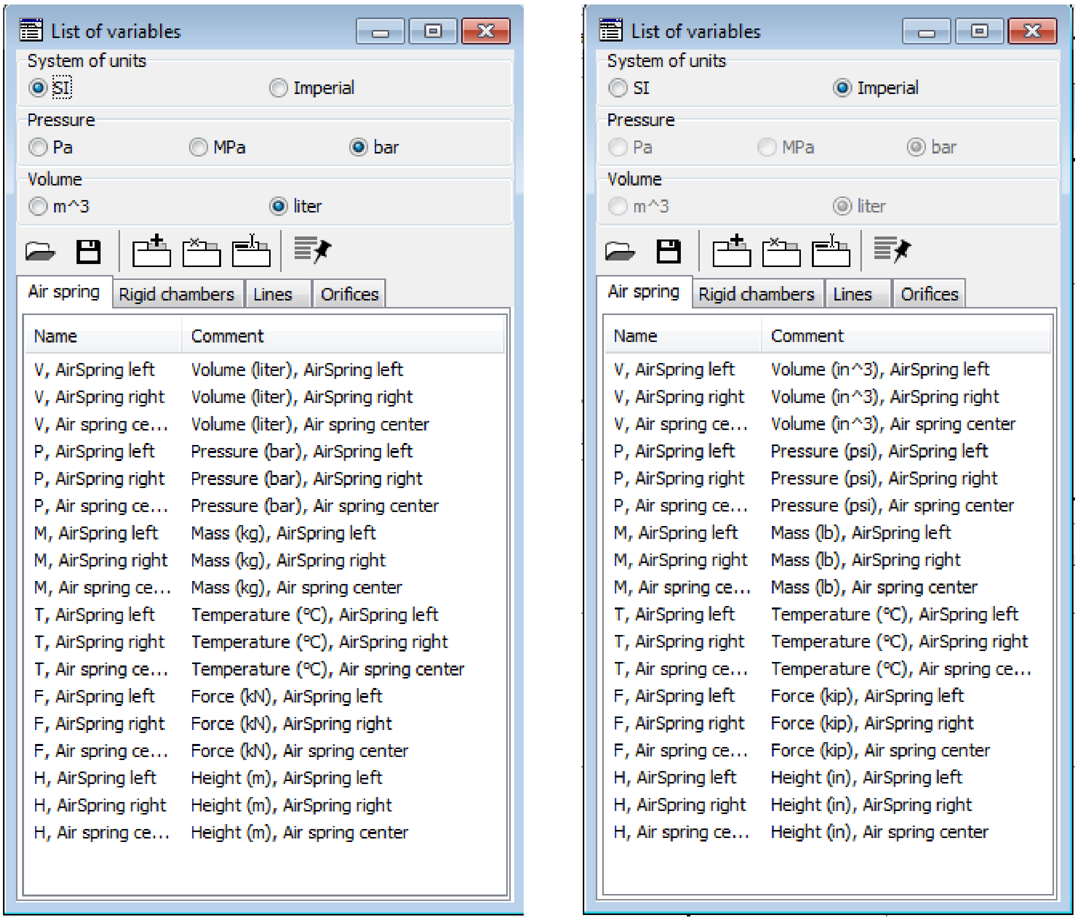


Figure 31.52. List of pneumatic variables in SI and Imperial systems of units

Table 2. Data units

System of units	Height	Force	Pressure	Volume
SI	m	N, kN	Pa, MPa, bar	m^3, liter
Imperial	in	kip	psi	in^3

31.4. Tests and examples

In this section we consider tests of PS models created taking into account publications of other authors. We selected a number of papers, which contain full information on PS as well as experimental results. Basing on these data we developed UM models of PS and compared our simulation results with experimental results from the papers.

31.4.1. Charge and discharge of tank

31.4.1.1. Case 1: Discharge

The model

{UM Data}\SAMPLES\ Pneumatics\Discharge

corresponds to paper [15]. Experimental data for a tank discharge are presented in this paper. The sonic conductance and the critical pressure ratio are given for two valves (V1, V2). Comparison of UM simulations (solid lines) and experimental data is shown in Figure 31.53. Plots correspond to the pressure fall verses time. Parameters of PS are stored in the files

Valve1.psc

Valve2.psc

located in the model directory.

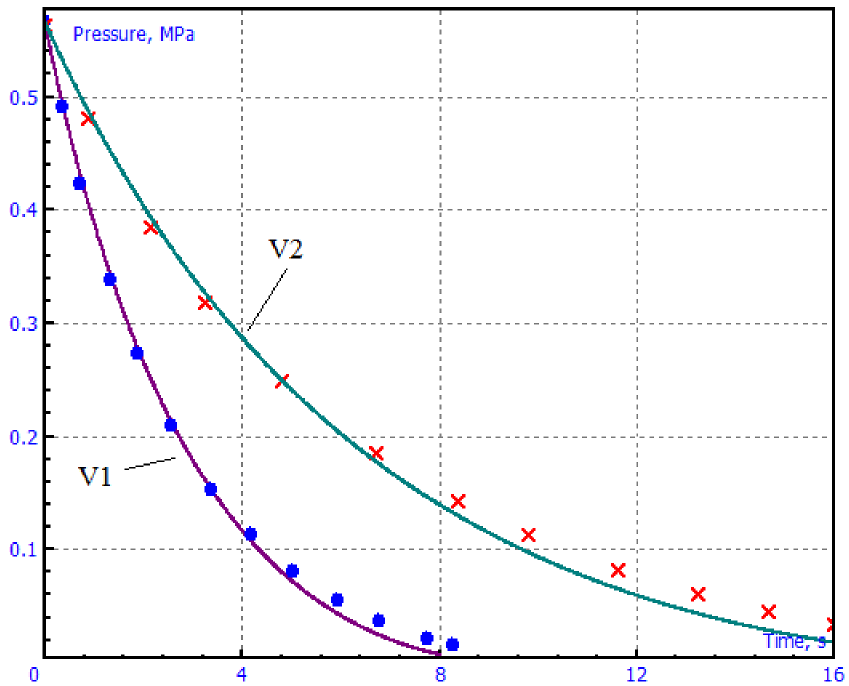


Figure 31.53. Comparison of simulation results and measurements for tank discharge

The model of PS contains two rigid chambers, Figure 31.54: the first one corresponds to the 10.6 l tank, and the second one with a large volume corresponds to the environment (constant pressure). An orifice connecting the chambers corresponds to a valve. The ISO 6358 standard is used for the valve model; the orifice diameter value is ignored in this test.

Thus, this example illustrates

- a good correlation of simulation and experiment;
- a source of a constant pressure as a rigid chamber with large volume.

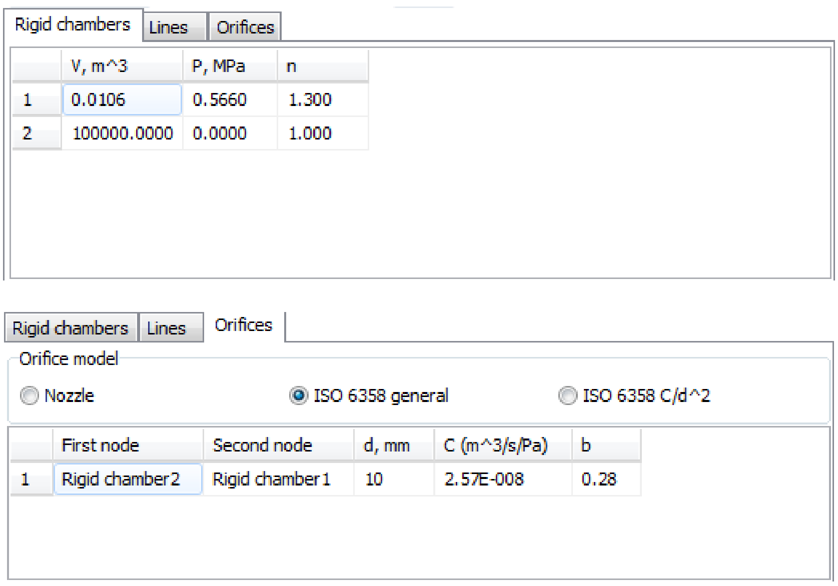


Figure 31.54. Rigid chambers and connecting orifice

How do repeat the test?

1. Open the model {UM Data}\SAMPLES\ Pneumatics\Discharge in UM Simulation.
2. Load the experimental results into the graphical window from two text files
P Valve 1 (experiment).txt,
P Valve 2 (experiment).txt
with the popup menu command, Figure 31.55.
3. Run simulation.

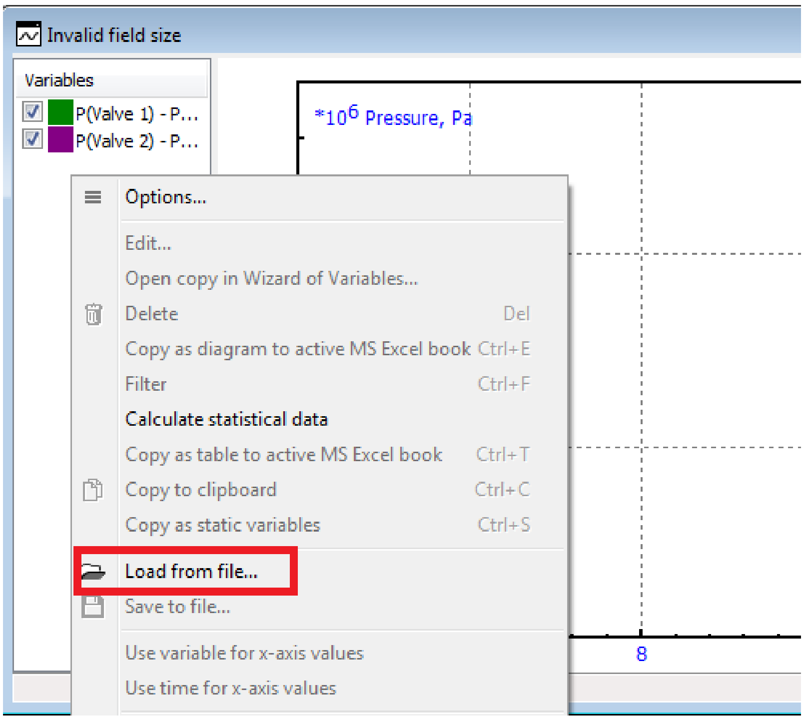


Figure 31.55. Appending plots from files

31.4.1.2. Case 2: Charge and discharge

The next test was taken from paper [16]. There the authors consider both the charge and discharge of a tank for several pressures. UM Model is

{UM Data}\SAMPLES\ Pneumatics\Charge and discharge.

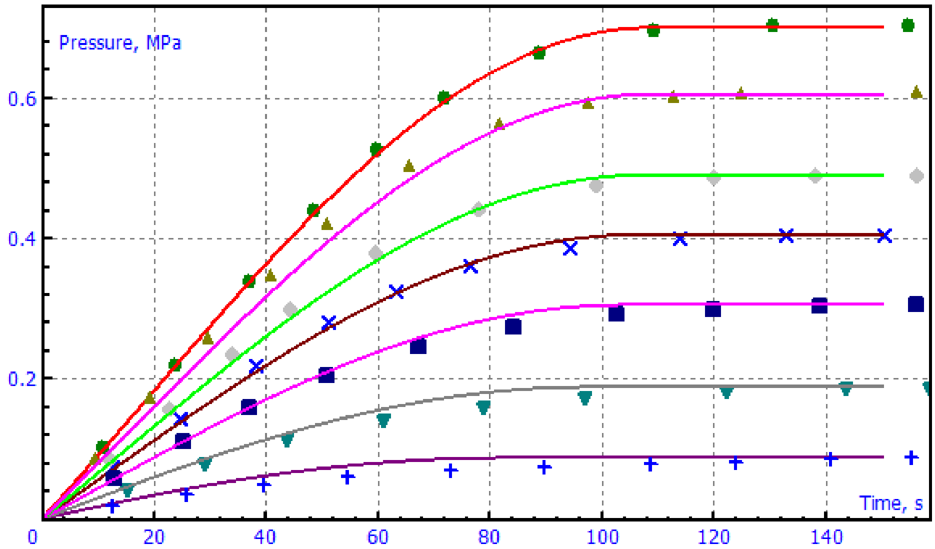


Figure 31.56. Comparison of simulation of tank charge process with measured data (markers) for different pressure values

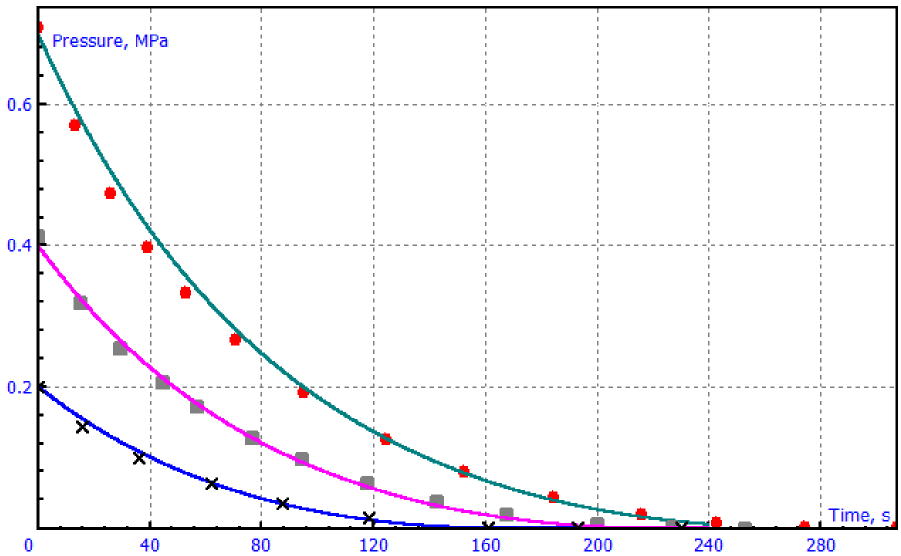


Figure 31.57. Comparison of simulation of tank discharge process with measured data (markers) for different pressure values

UM simulation results are compared with measurements in Figure 31.56 and Figure 31.57.

How do repeat the test?

1. Open the model {UM Data}\SAMPLES\ Pneumatics\Charge and discharge in UM Simulation.

2. Following Figure 31.55, load the experimental results into the graphical windows from text files:

- in the window with **P (Discharge)** variable:

Discharge 2bar.txt
Discharge 4bar.txt
Discharge 7bar.txt

- in the window with **P (Charge)** variable:

Charge 1bar.txt
Charge 1.9bar.txt
Charge 3bar.txt
Charge 4bar.txt
Charge 5bar.txt
Charge 6bar.txt
Charge 7bar.txt

3. Set the desired discharge value of pressure to the small chamber (pneumatic system **Discharge**) and the charge value for the large chamber (pneumatic system **Charge**), Figure 31.58.

4. Run simulation.

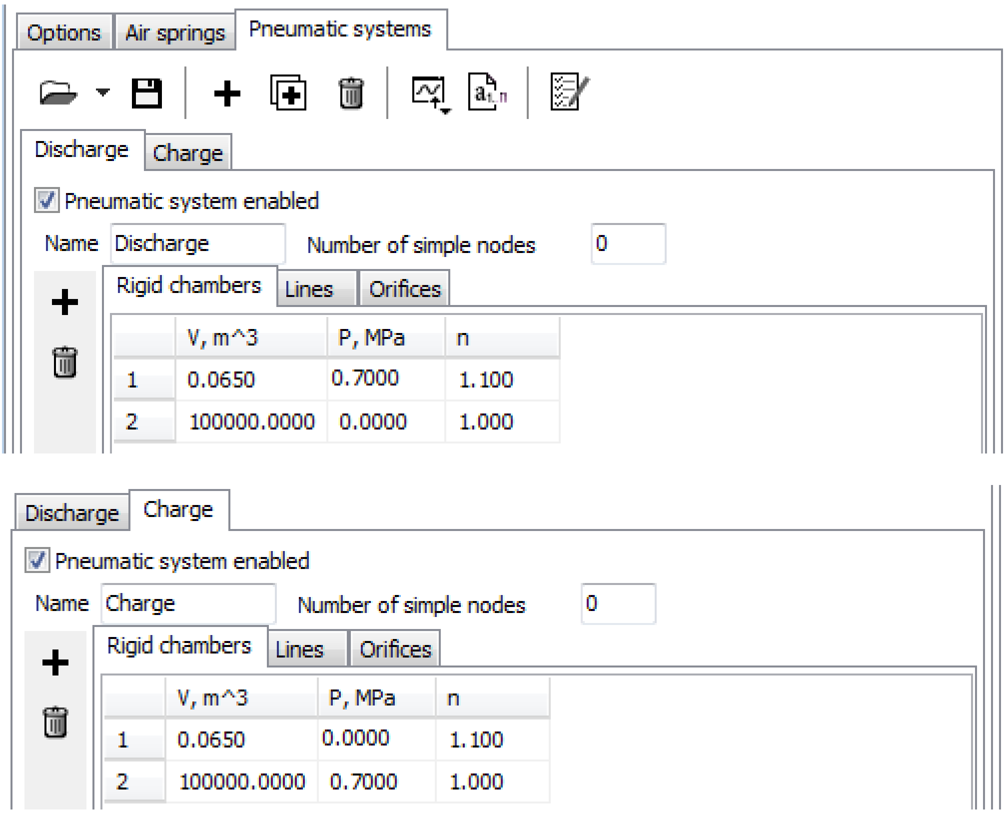


Figure 31.58. Pneumatic systems for charge and discharge simulation

31.4.2. Dynamic stiffness and damping

31.4.2.1. Case 1: Air spring connected by pipeline with auxiliary chamber

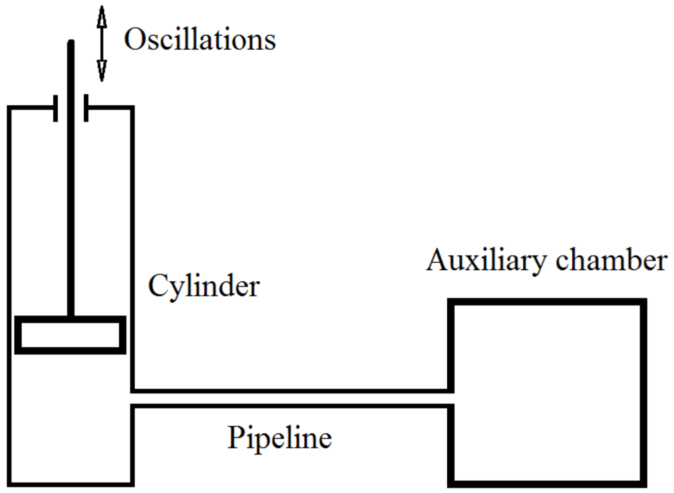


Figure 31.59. Scheme of experiment for study of pipeline influence on dynamic stiffness and damping of air spring with auxiliary chamber

The study corresponds to an experiments described in paper [17]. A pneumatic cylinder with 100mm diameter is considered as an air spring. The cylinder is connected with an auxiliary chamber by a pipeline. Harmonic oscillations are applied to the cylinder rod, and the force applied to the rod is measured to estimate the influence of the pipeline on the dynamic stiffness and damping of the air spring, Figure 31.59.

UM Model is

{UM Data}\SAMPLES\ Pneumatics\Dynamic pipeline.

Test parameters:

Cylinder volume at equilibrium: 1.1 liter;

Volume of auxiliary chamber: 2.2 liter;

Pipeline length and diameter: L=1.5m, D=7.5mm;

Static value of absolute pressure: 552 kPa.

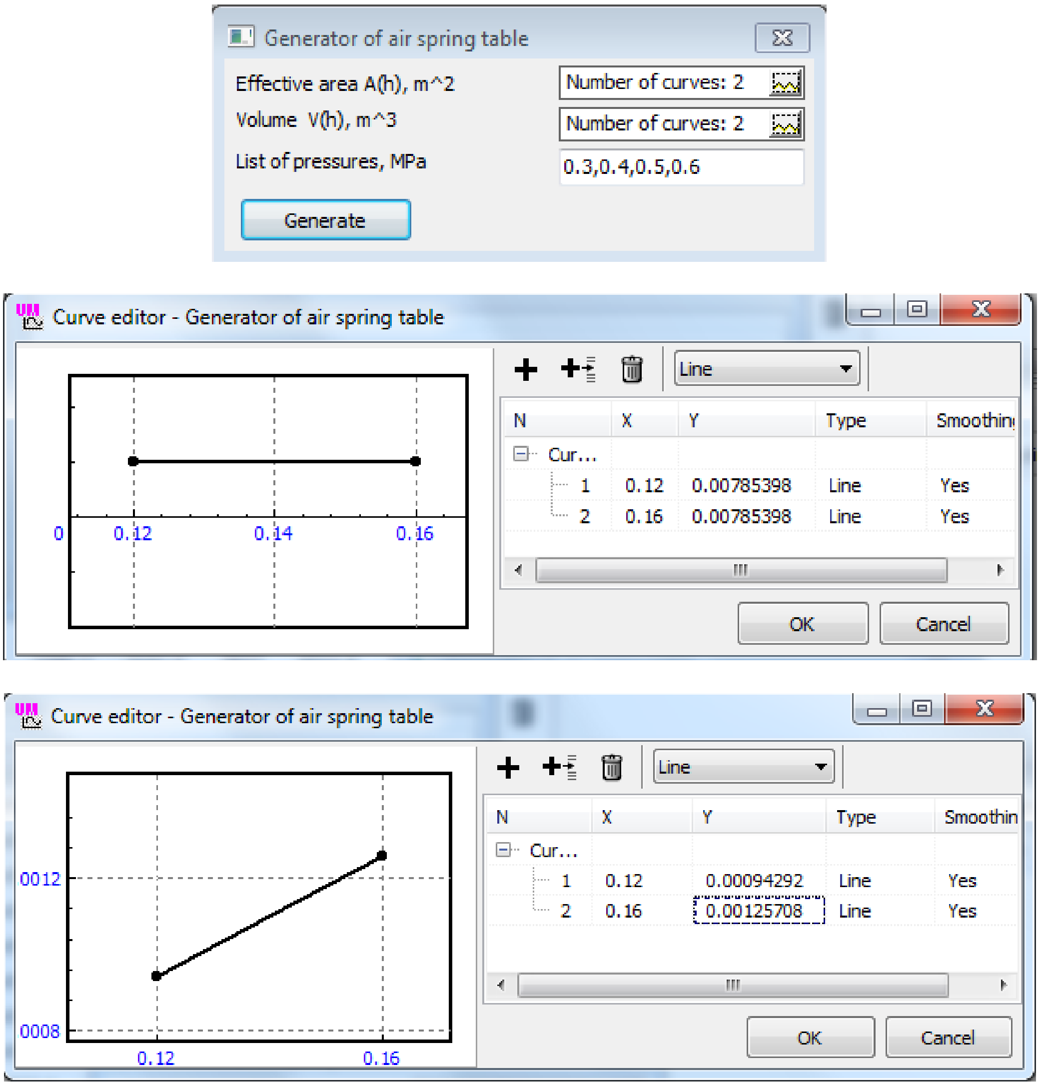


Figure 31.60. Generation of tabular model of cylinder as air spring

The model of the pneumatic cylinder as an air spring is created with the Generator of air spring table, Sect. 31.2.3.1.3.5. *Creating tabular data by effective area*. The effective area is constant

$$A_e = \pi D^2/4 = 0.00785398 \text{ m}^2$$

and the volume is the linear function of the height

$$V_{as} = A_e h$$

Description of the corresponding AS model is presented in Figure 31.60. The tabular model is generated and saved in the file "Pneumatic cylinder D100.ast".

Consider useful values of the AS stiffness constant for very low and very high frequencies of excitation. We use the expression for the air spring stiffness

$$K = \left|\frac{dF}{dh}\right| = \left|\frac{dp}{dh}\right| A_e$$

Assuming the polytropic process, the derivative dp/dh is computed from the relation

$$\frac{dpV^n}{dh} = \frac{dp}{dh}V^n + npV^{n-1}\frac{dV}{dh} = \frac{dp}{dh}V^n + npV^{n-1}A_e = 0,$$

$$\left|\frac{dp}{dh}\right| = \frac{npA_e}{V}$$

which results in

$$K = \frac{npA_e{}^2}{V}.$$

Using this result, we can compute the stiffness for low frequency, where the aggregate volume of the AS and the auxiliary chamber is substituted in the formula

$$K_L = 14.2 \text{ N/mm}.$$

as well as the stiffness constant for the high frequencies, where only the AS volume should be taken into account

$$K_H = 42.7 \text{ N/mm}$$

The computations were done for the polytropic index n=1.38.

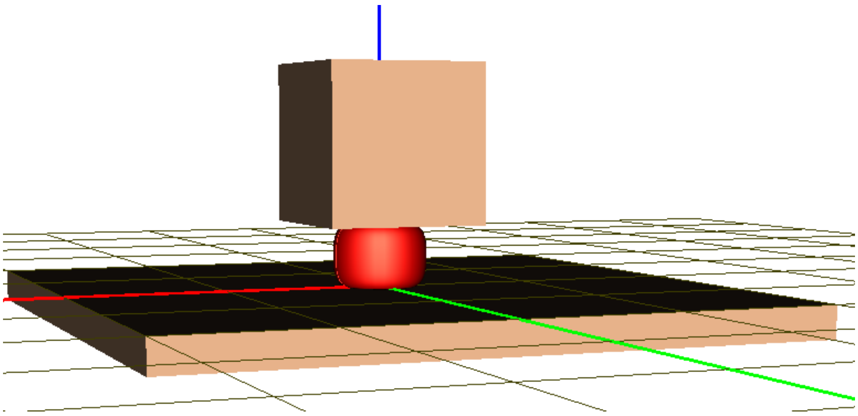


Figure 31.61. Mechanicl part of model "Dynamic pipeline"

Let us consider now simulation of the model with UM. The mechanical part of the model "Dynamic pipeline" contains two bodies, connected by the AS force element:

- Base - the lower body, which can oscillate with a constant or variable frequency;
- Body - the upper body, which is fixed.

So, the oscillation of the Base leads to the corresponding change of the AS height.

The jBase joint implements the oscillations of the base with the gliding frequency

$$f = f_0 + \epsilon t \text{ Hz},$$

see Figure 31.62. The following identifiers parameterize the expression:

- f0 - start frequency;
- eps - frequency rate;
- ampl - amplitude of oscillations.

If eps=0, the harmonic oscillations with the frequency f_0 take place.

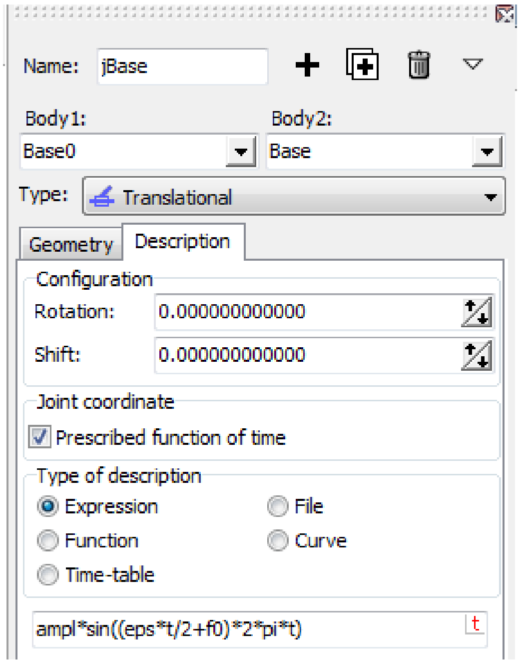


Figure 31.62. Oscillation with gliding frequency

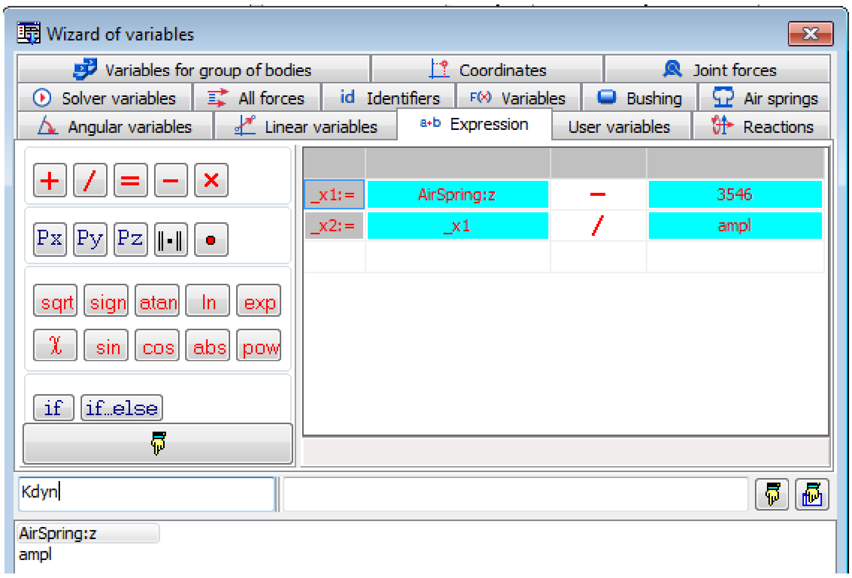


Figure 31.63. Variable for evaluation of dynamic stiffness of AS

With this model we compare simulated and measured dynamic stiffness of the air spring. With this purpose, the variable *Kdyn* is created, which is the centralized air spring force divided by the amplitude of oscillations, Figure 31.63. The enveloping curve on the plot of this variable versus the frequency corresponds to the dynamic stiffness of the AS.

Parameters of the line model are shown in Figure 31.64. It is worth to draw attention to the type of the **Time domain** line model, and to the e/d = 0.01 value for the relative pipe roughness. It is interesting that neither polytropic index nor the number of segments affect considerably on simulation results shown below.

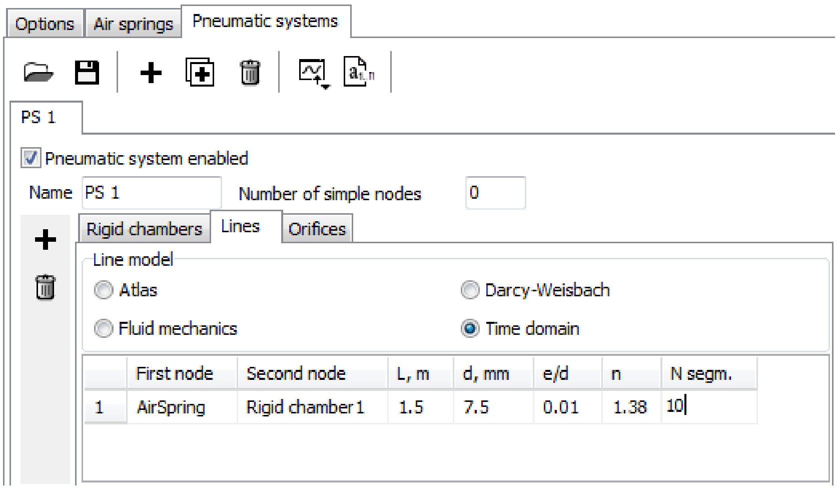


Figure 31.64. Parameters of line model

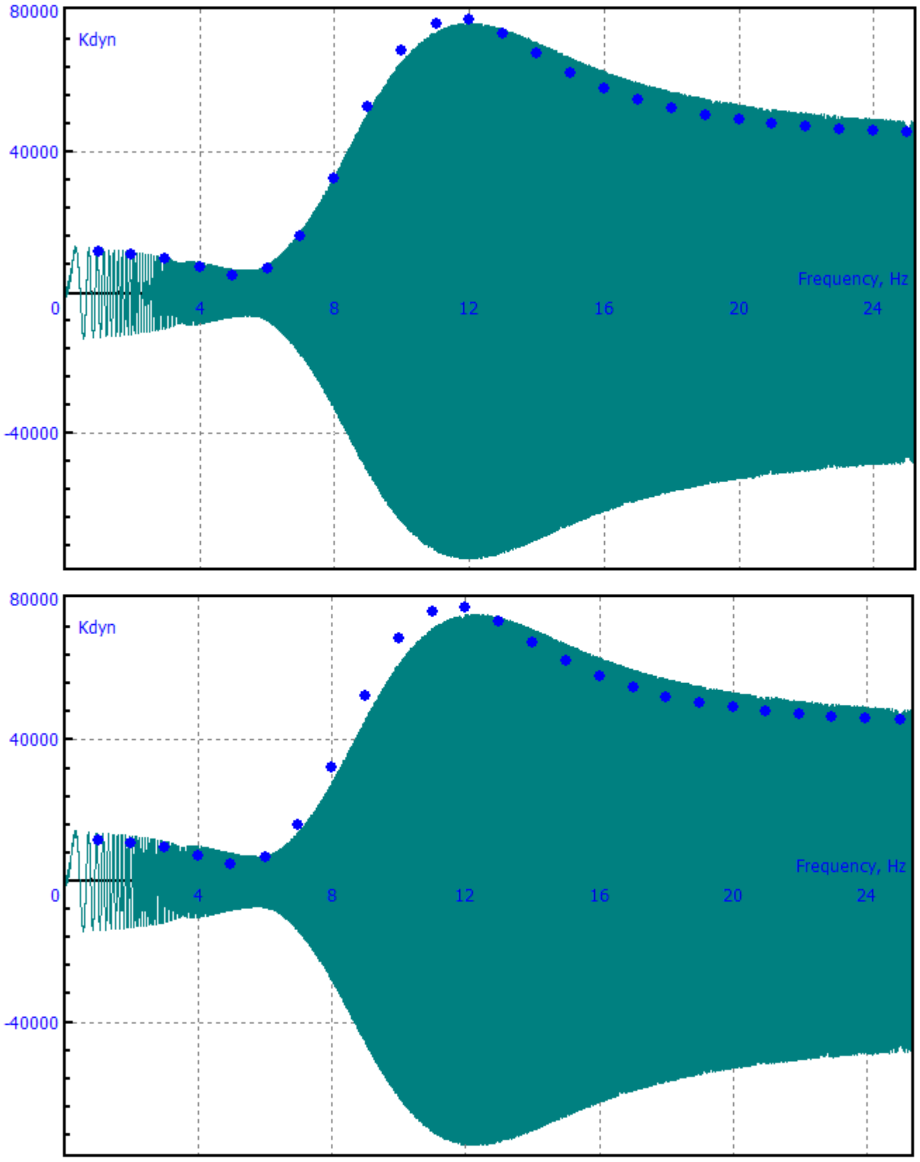


Figure 31.65. Dynamic stiffness vs. frequency: comparison of simulation and experiment: amplitude 1mm, polytropic index for auxiliary chamber n=1.2 for the upper and n=1.38 for the lower plot.

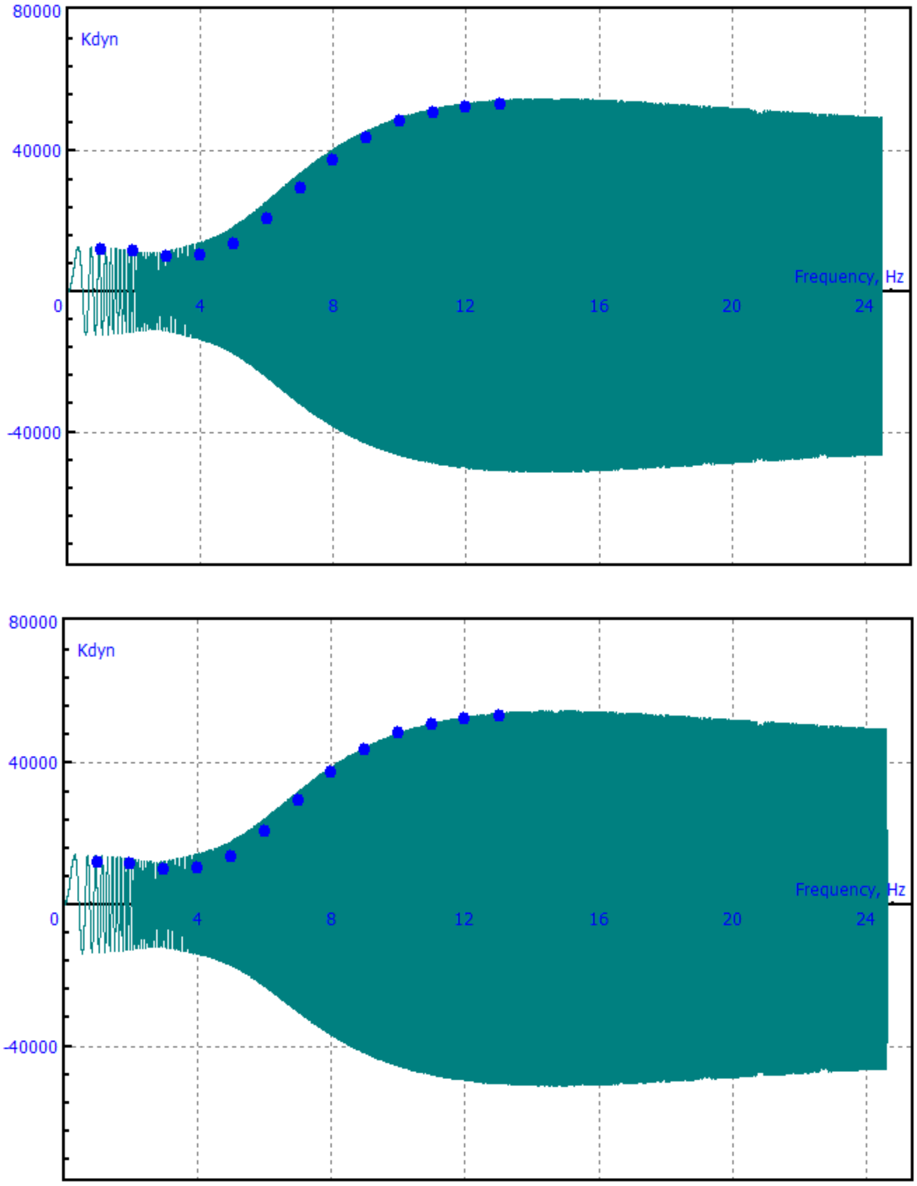


Figure 31.66. Dynamic stiffness vs. frequency: comparison of simulation and experiment: amplitude 5mm, polytropic index for auxiliary chamber n=1.2 for the upper and n=1.38 for the lower plot.

Comparison of UM simulation results on dynamic stiffness with the experimental data from paper [17] are shown in Figure 31.65, Figure 31.66. To fit better the experimental data, we recommend the standard polytropic index n=1.38 for the air spring volume and the lower value 1.2 for the auxiliary chamber. Anyway, coincidence seems to be very good.

The experimental results shown in figures by markers are located in the files

1mm (experiment).txt,

5mm (experiment).txt

and can be used for repeating tests with the UM model. The excitation amplitude can be changed in the list of variables, Figure 31.67.

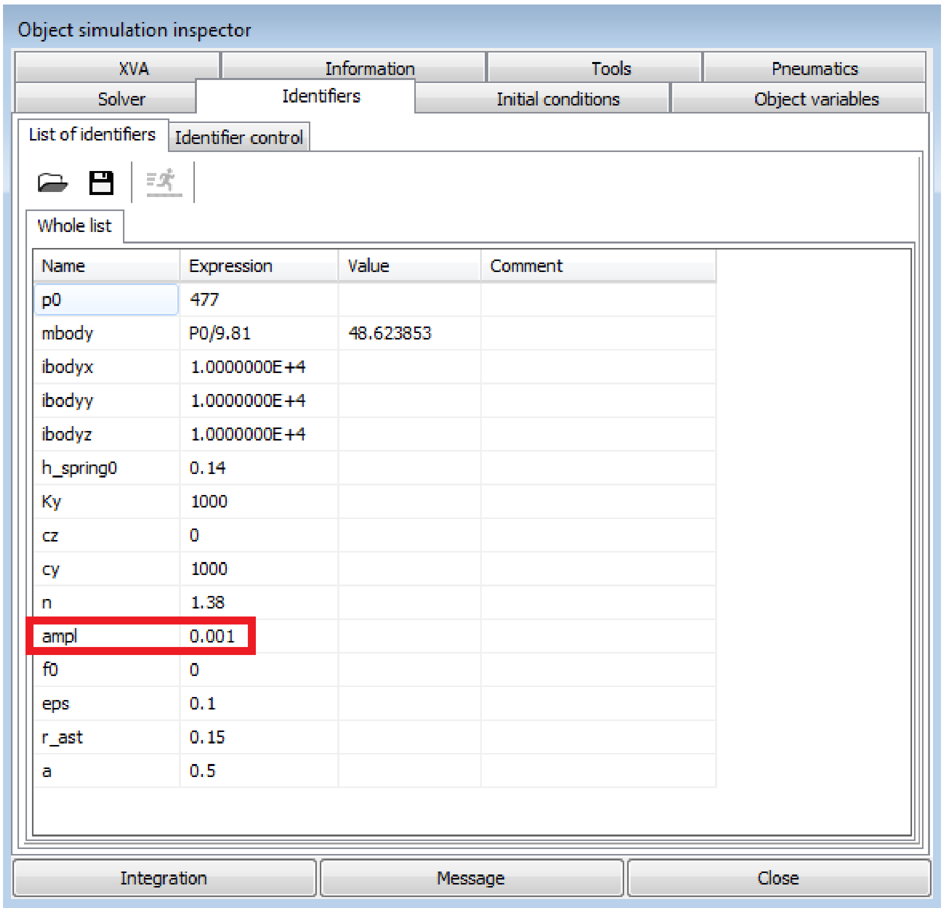


Figure 31.67. Change of excitation amplitude

Finally, comparison of stationary and time domain models in Figure 31.68 confirms that the stationary model of a long pipe can be used for the slow processes only.

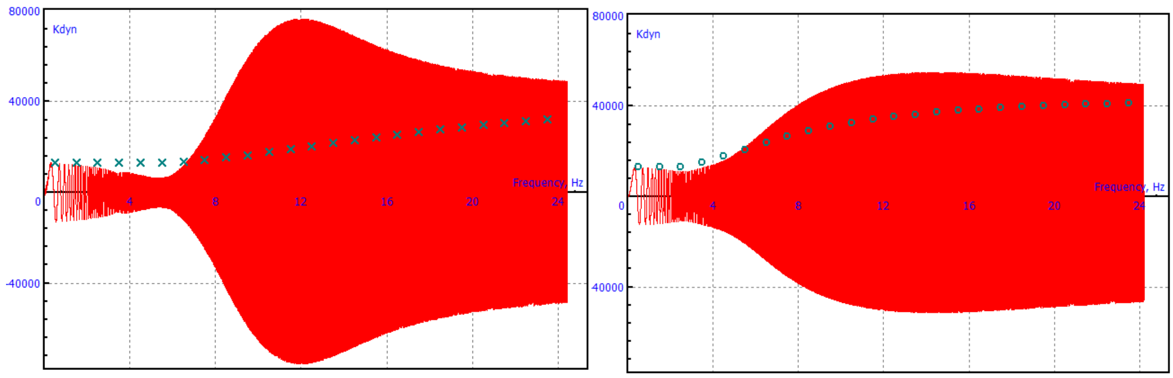


Figure 31.68. Comparison of stationary (marker) and time domain models

31.4.2.2. Case 2: Air spring connected by orifice with auxiliary chamber

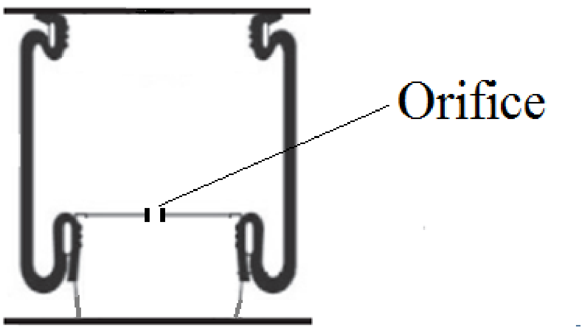
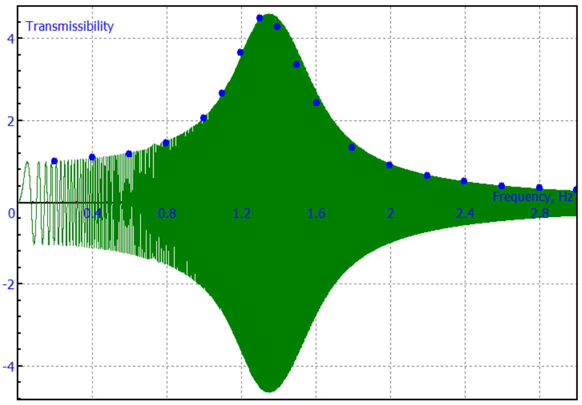


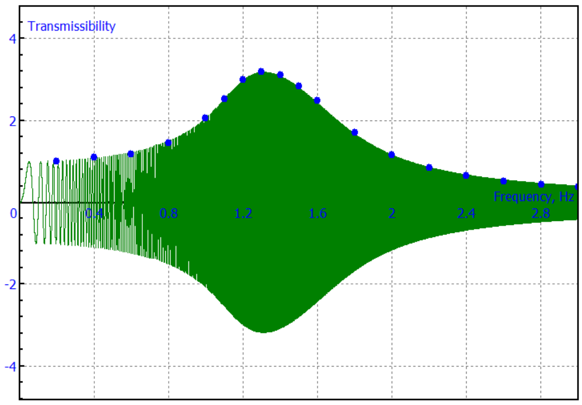
Figure 31.69. Air spring and auxiliary chamber

The test corresponds to the experimental data presented in paper [18]. In the test, an air spring is connected with an auxiliary chamber by an orifice, Figure 31.69. The paper includes the dependences of the effective area and volume of the air spring on its height, which allow us to develop the AS model, Sect. 31.2.3.1.3.5 *Creating tabular data by effective area*. The experiment consists of the harmonic excitation of the air spring foundation and the measurement of the dead weight oscillations. The UM model is

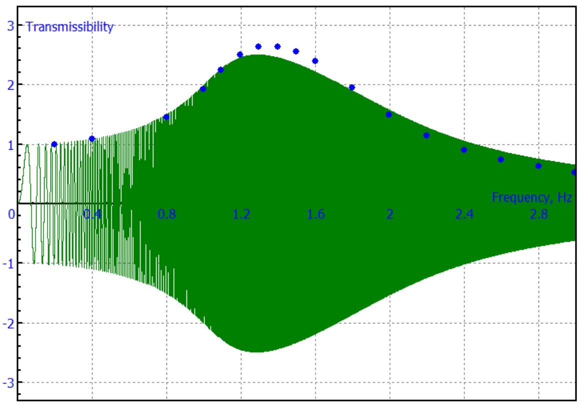
{UM Data}\SAMPLES\ Pneumatics\Orifice test



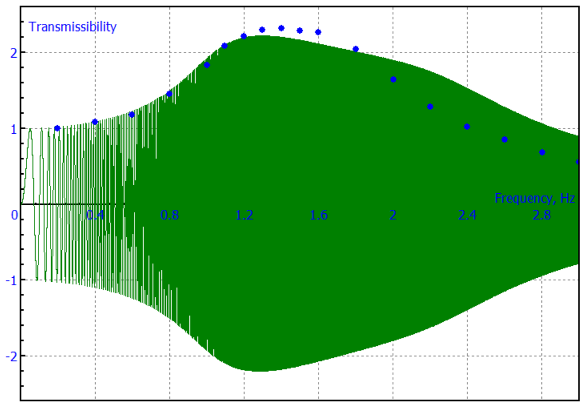
a = 1mm



a = 2.5mm



a = 5mm



a = 7.5mm

Figure 31.70. Comparison of simulation and experimental results

We compared simulation results for the static absolute pressure 280 kPa with experiments obtained in [18] for different excitation amplitudes, Figure 31.70. The plots show dependencies of the transmissibility on the frequency for different excitation amplitudes *a*. The transmissibility is the ratio of the response amplitude to the excitation amplitude *a*. The "Nozzle" orifice model is used.

The experimental results from paper [18] shown in figures by markers are located in the files

Transmissibility 1mm.txt,

Transmissibility 2.5mm.txt,

Transmissibility 5mm.txt,

Transmissibility 7.5mm.txt,

and can be used for repeating tests with the UM model. The excitation amplitude can be changed in the list of variables similar to Figure 31.67.

31.4.3. Models with air springs

31.4.3.1. Testing stand with 3 air springs

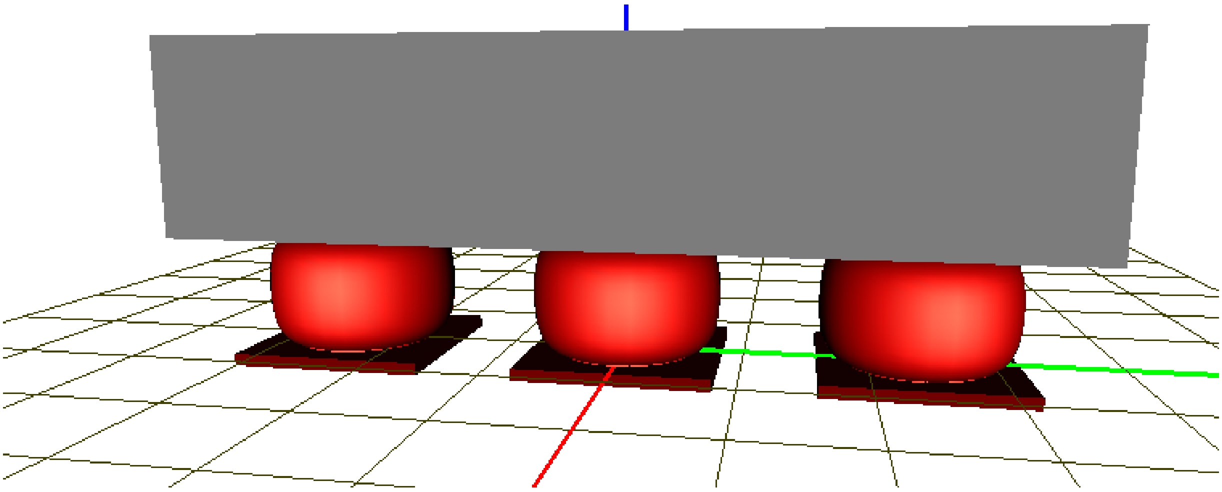


Figure 31.71. Testing stand with three AS

The stand model is located in
{UM Data}\SAMPLES\ Pneumatics\test_3as
The model includes three air springs supporting a rigid body, which has 3 d.o.f. in the vertical plane, Figure 31.71. Positions of three plates under each of the AS are parameterized by the identifiers *z_as*1, *z_as*2, *z_as*3, Figure 31.73.

The model has been developed for testing a suspension with air springs interconnected by pipelines, Figure 31.72. The pipelines are connected by a T-junction, which is modeled in UM PS by a simple node, Sect. 31.2.4 *Simple nodes* (**Node 1** in Figure 31.38).

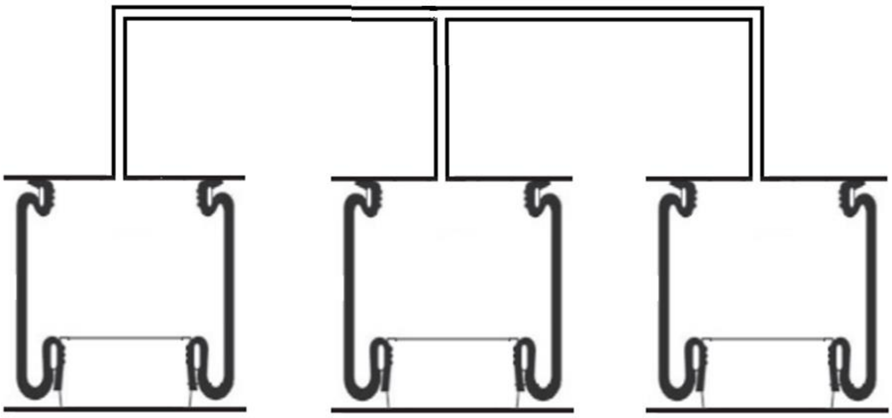


Figure 31.72. Interconnected air springs

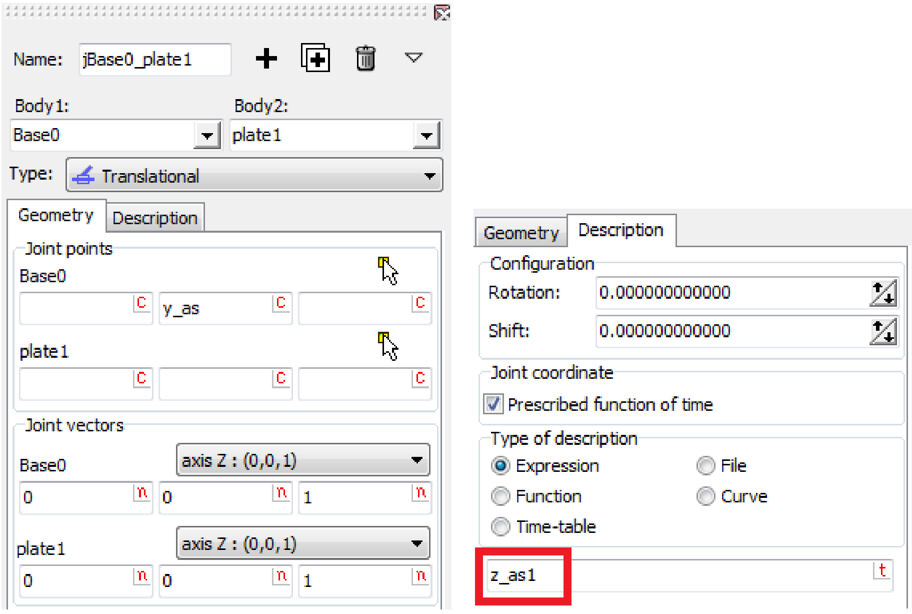


Figure 31.73. Joint for Plate1

The user can describe the motion of the plates under each of the AS with the **Identifier control** tool, Figure 31.74. As examples, we prepared two excitations for the plate1: step and harmonic functions, Figure 31.75, Figure 31.76.

To create a step function of time, we used the curve editor, see Figure 31.75. The harmonic function was made with the **Wizard of variables** and dragged into the **Assigned variable** box of the **Identifier control** window, Figure 31.76. We used identifiers for the excitations frequency (*freq*) and amplitude (*ampl*), so the user can change their values in the list of identifiers of the model.

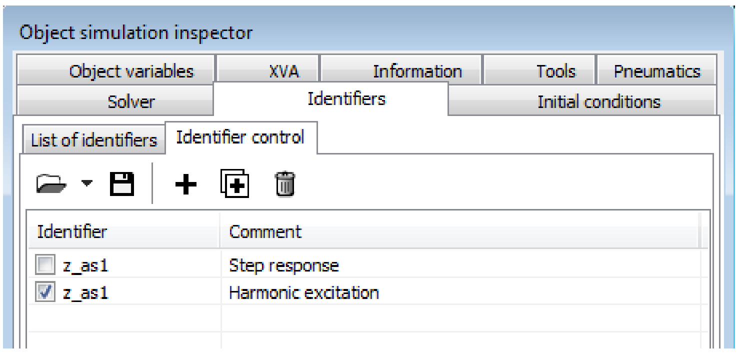


Figure 31.74. Examples of identifier control for vertical position of Plate1

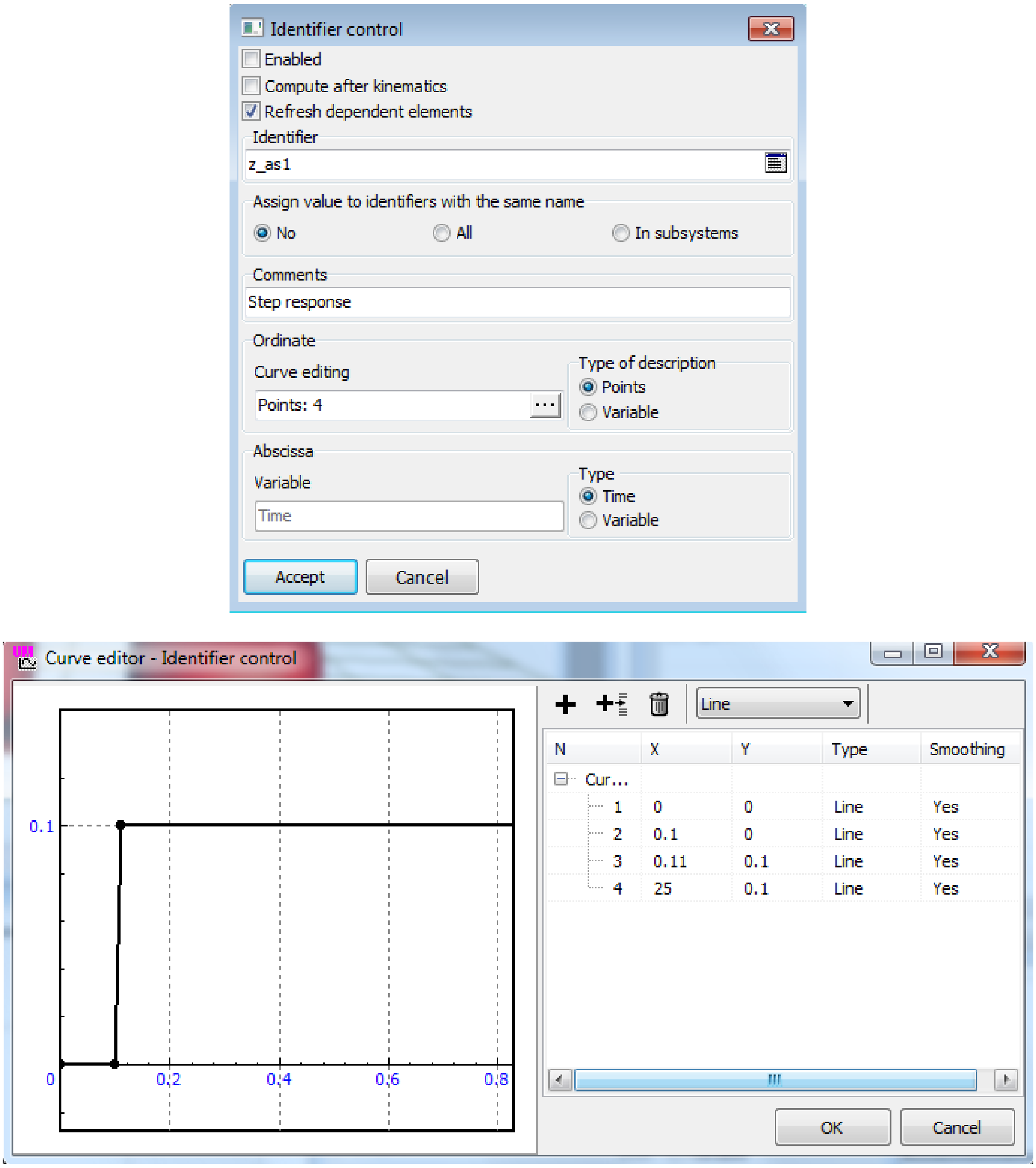
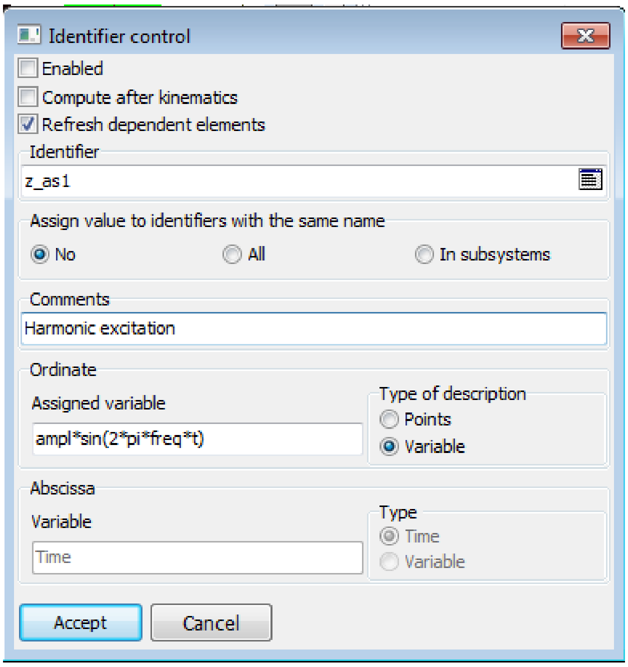


Figure 31.75. Step function for identifier z_as1



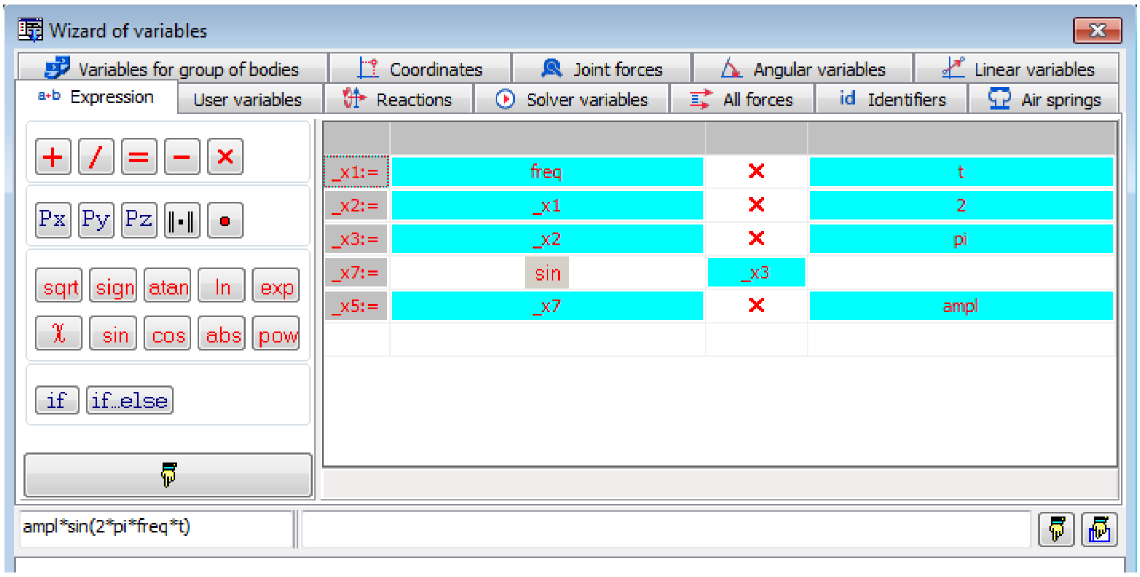


Figure 31.76. Harmonic function for identifier control

31.4.3.2. Testing stand with 6 air springs

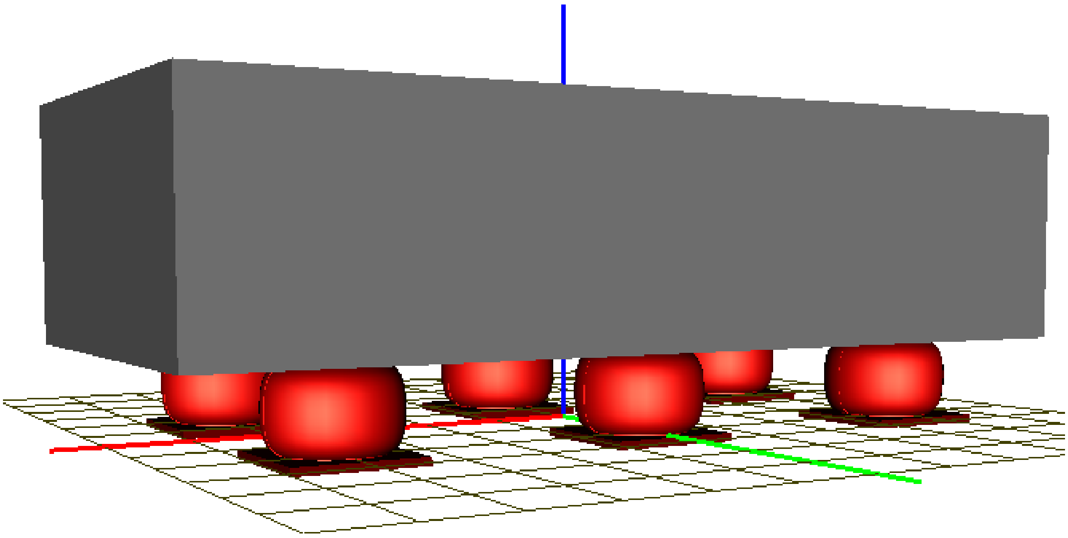


Figure 31.77. Testing stand with six AS

The stand model is located in
{UM Data}\SAMPLES\ Pneumatics\test_6as

The model includes six air springs supporting a rigid body, which has 6 d.o.f., Figure 31.71. Positions of six plates under each of the AS are parameterized by the identifiers *z_as*1*l*, *z_as*2*l*, *z_as*3*l*, *z_as*1*r*, *z_as*2*r*, *z_as*3*r*.

The model has been developed for testing suspensions with air springs interconnected by pipelines.

We developed two types of AS connections.

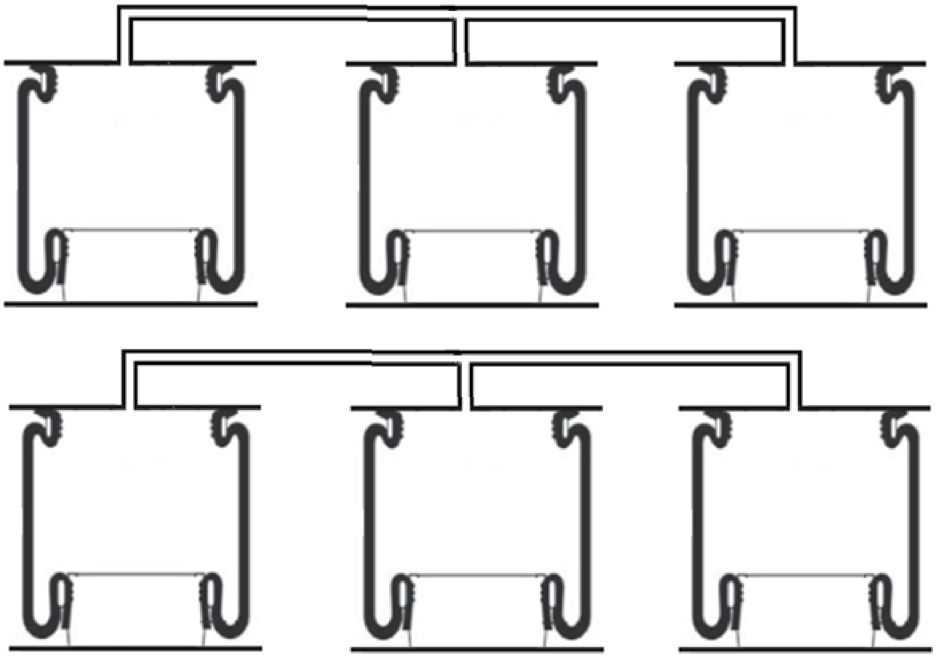


Figure 31.78. PS 1 scheme

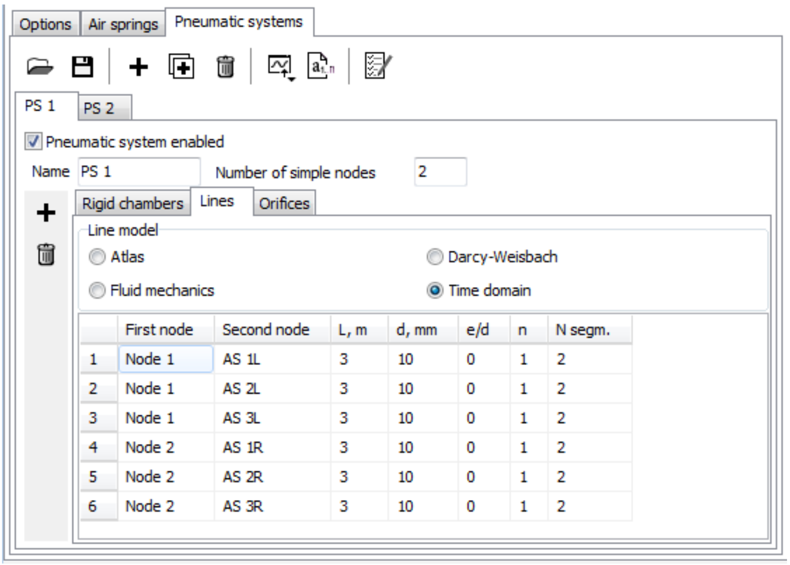


Figure 31.79. PS 1 connections

PS1, Figure 31.78: Left and rights AS are connected independently by two T-junctions, Node 1, Node 2 in Figure 31.79.

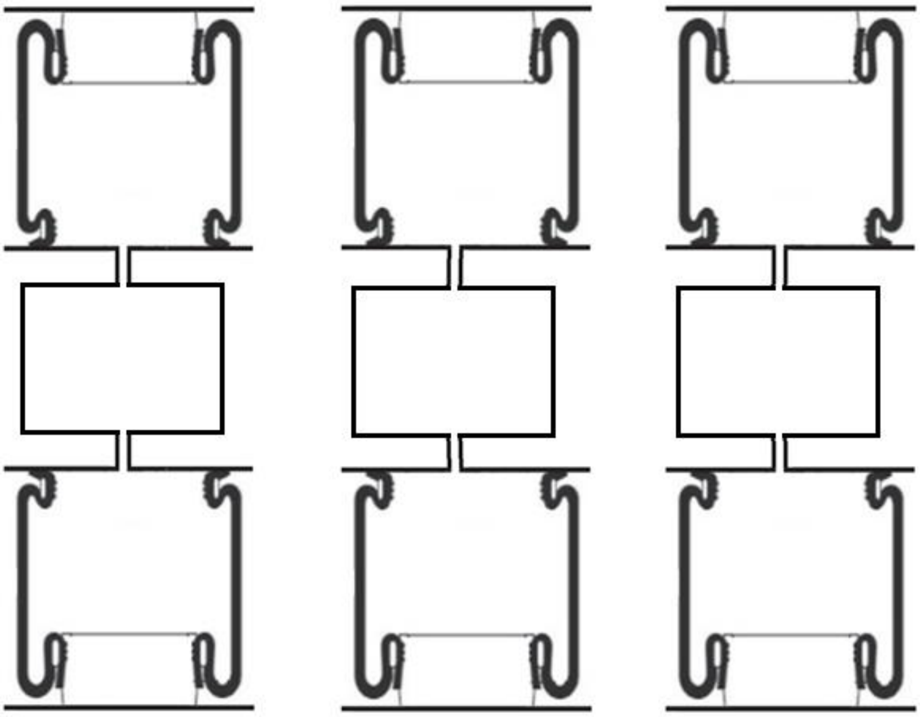


Figure 31.80. PS 2 scheme

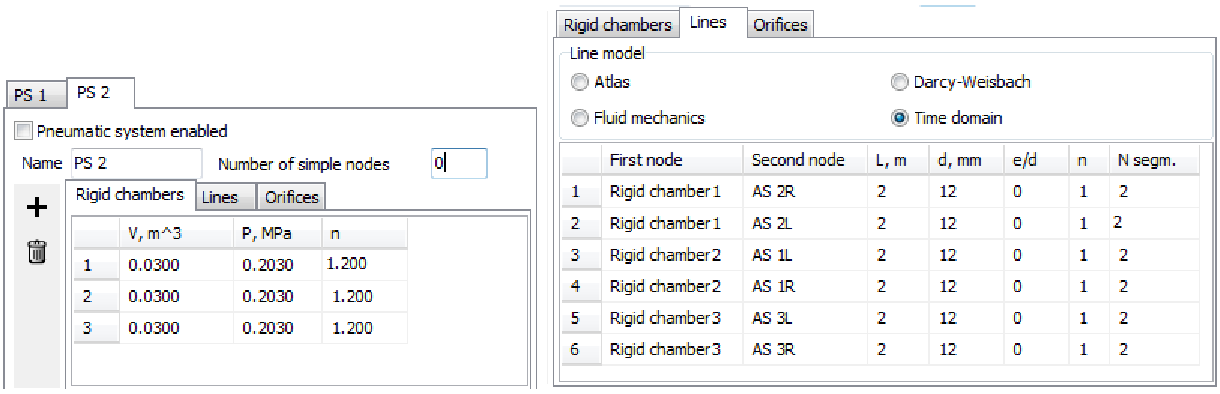


Figure 31.81. PS 2 connections

PS2, Figure 31.80: Left and rights AS are connected independently to three rigid chambers, Figure 31.81.

The stand excitation functions are created as variables of time, Figure 31.76, Figure 31.82. The variables are stored in the *Excitation functions.var* file, which can be read by the user in a list of variables. Use the **Tools | List of variables** menu command to open the list form, and then the 💾 button to open the file, Figure 31.83.

The variable approximating the step function depends on two identifiers:

- *h_step* is the step height;

- *t_step* is the time constant specifying the rate of the function growth.

The use can use these and other variables to assign them to the identifier control like in Figure 31.84.

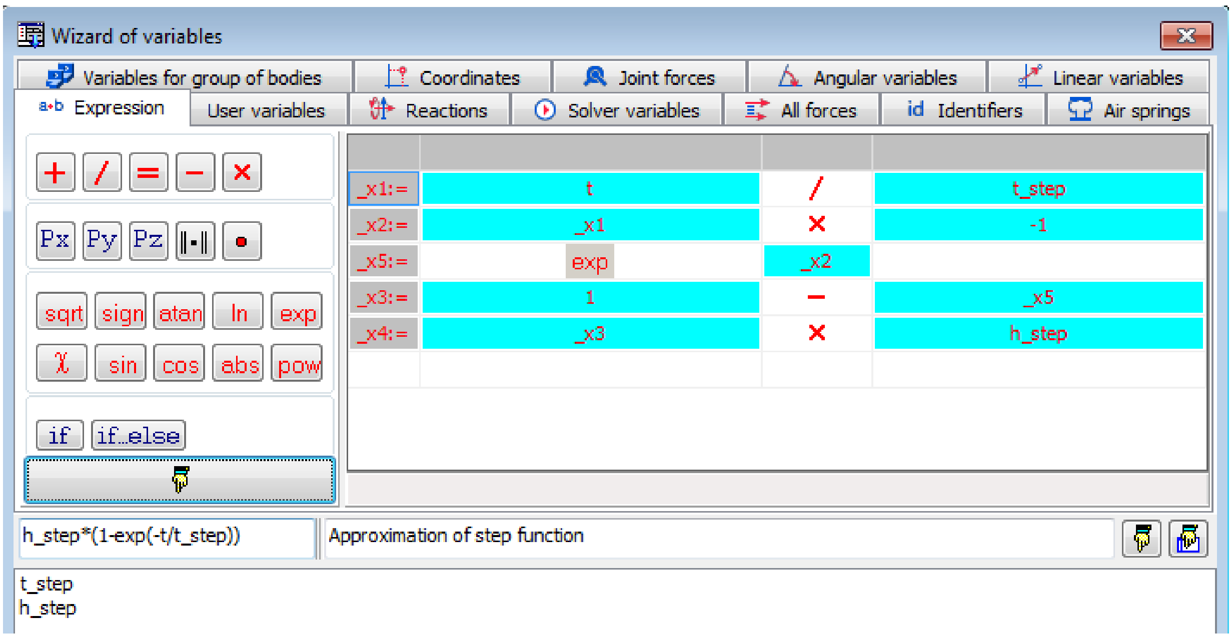


Figure 31.82. Variable approximating step function

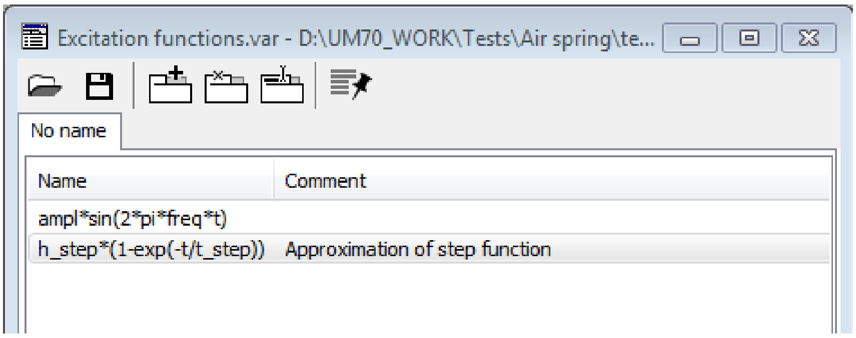


Figure 31.83. List of excitation variables

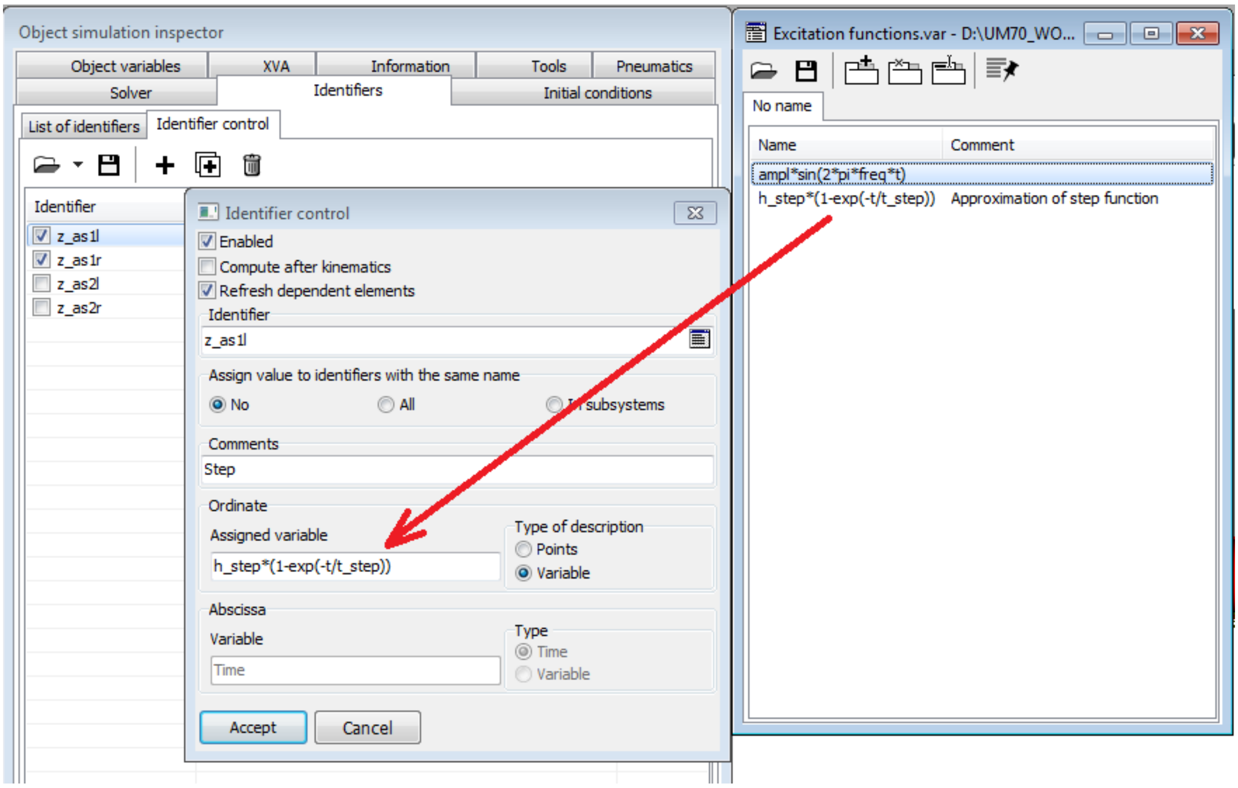


Figure 31.84. Assignment of excitation variables to identifier control

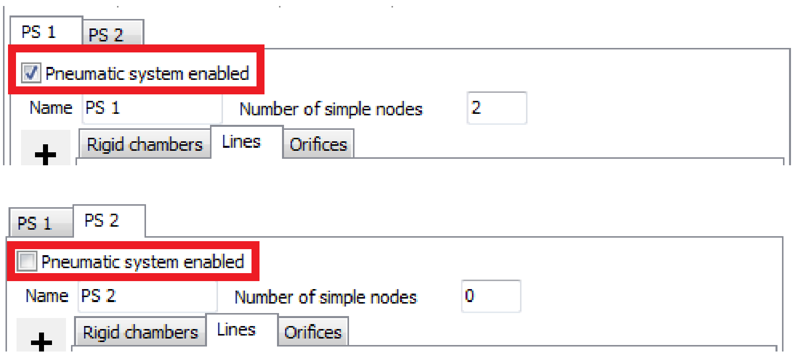


Figure 31.85. Enabled and disabled PS

The user can activate (enable) one of two PS before simulation start, Figure 31.85. If none of the PS is enabled, the air springs are considered as independent ones.

31.4.3.3. Test model: High speed railway motor car

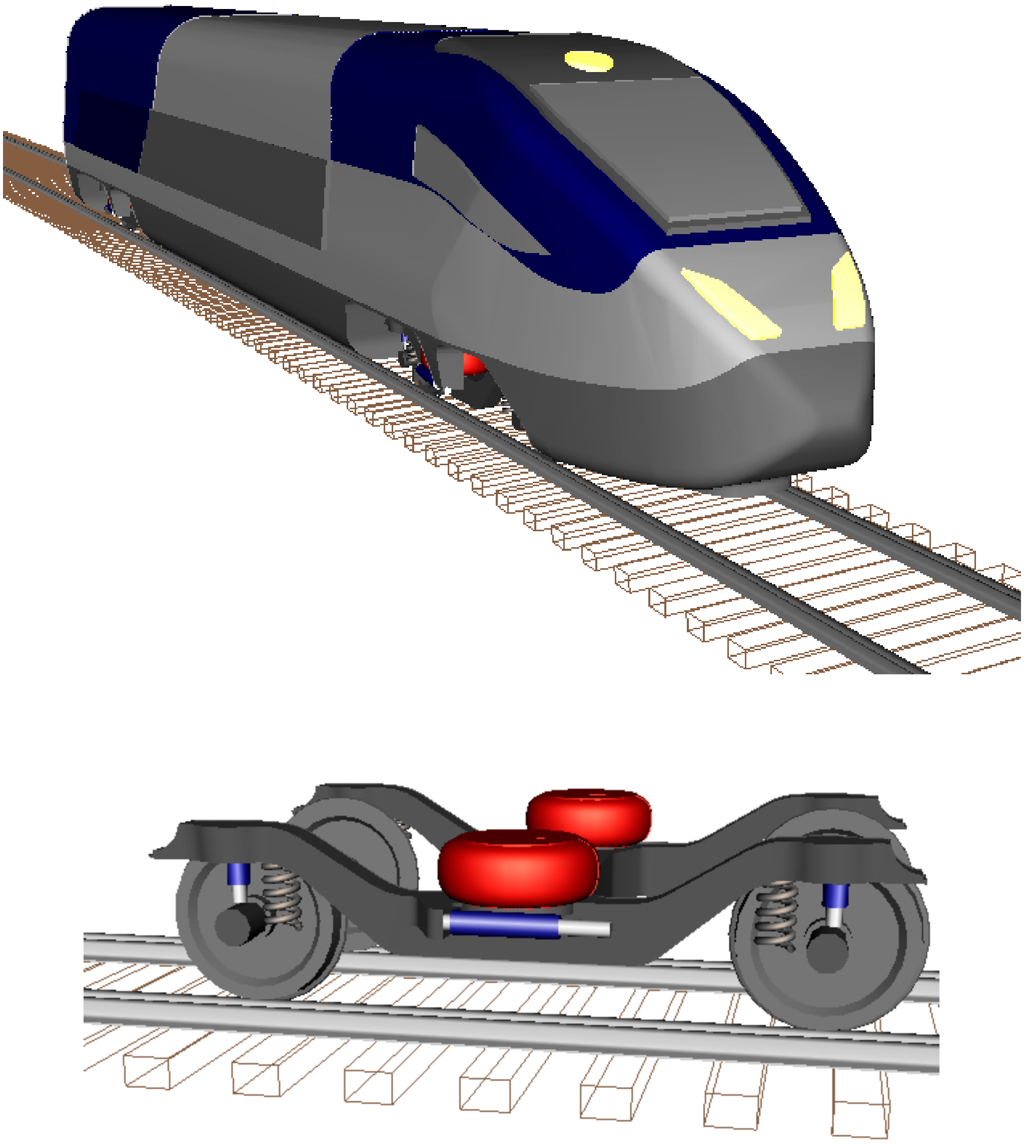


Figure 31.86. UM model of motorcar

The simplified model of a motorcar of a high speed train is located in
{UM Data}\SAMPLES\ Pneumatics\motorcar

The secondary suspension of the motorcar includes two air springs for each of the bogie, Figure 31.86. In the model we used data for the YI-FT 1710-38-324, ENIDINE Incorporated air spring [19], Figure 31.87.

With this model the user can compare simulation results for two cases:

Case 1: Isolated air spring

The pneumatic system is disabled, and vertical dampers in secondary suspension are enabled.

Case 2: Air springs connected with auxiliary chamber by pipelines, Figure 31.88.

The pneumatic system is enabled, and the secondary vertical dampers disabled.

To load the simulation cases, open the model and use one of the **File | Load configuration | Auxiliary chambers/Isolated air spring** menu commands.

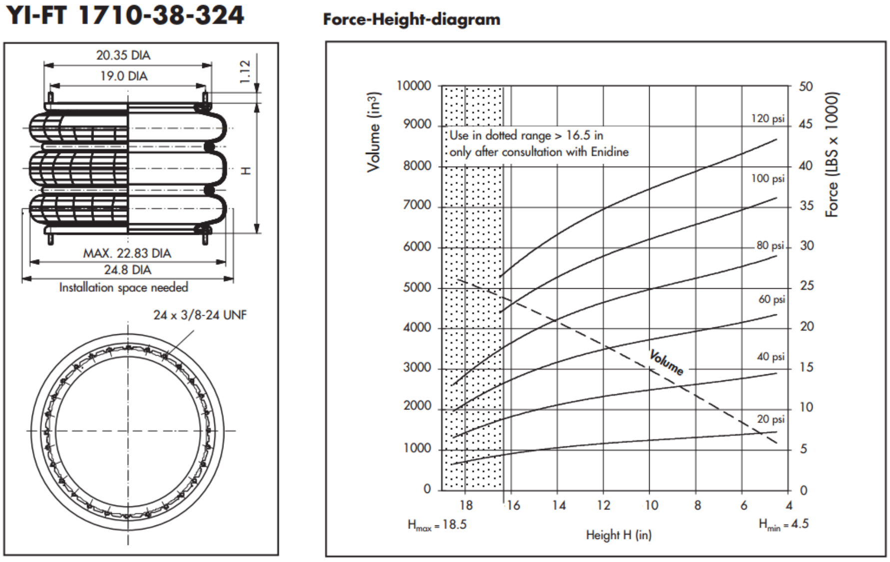


Figure 31.87. Force-height diagram for YI-FT 1710-38-324

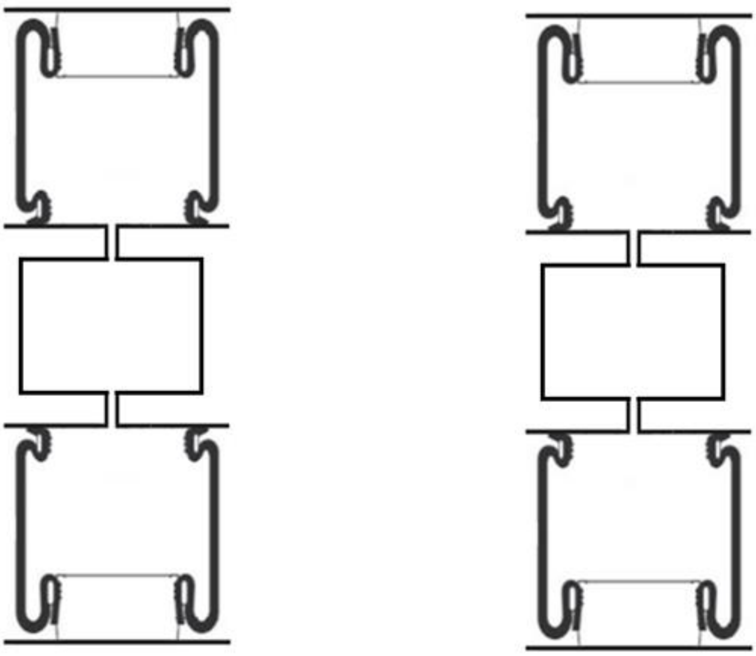


Figure 31.88. Air spring connections for bogies

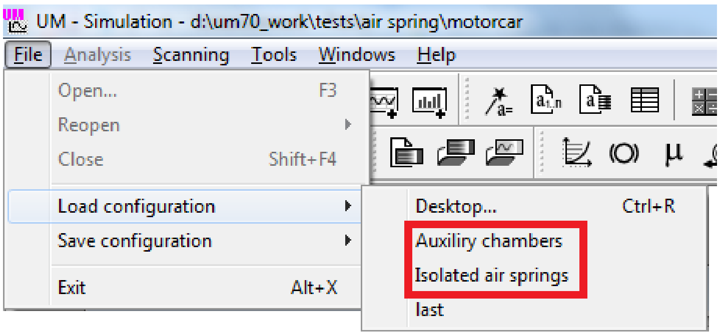


Figure 31.89. Selection of model configuration

31.5. Error messages

Here we consider messages related to preparing PS models and simulation process.

31.5.1. Errors in pneumatic system model

PS Model: Node(s) not assigned "Name"
One or two nodes are not assigned to a pipeline of an orifice.

PS Model: Equal nodes are assigned "Name"
Nodes assigned to a pipeline or an orifice must be different.

PS Model: Table model for air spring "Name" not assigned
Files with models should be assigned to all of the tabular air springs, Sect. 31.3.1 *Parameters of tabular air springs*.

PS Model: Air spring "Name" included in several enabled pneumatic systems
Tabular air spring can be included in **one** enabled PS only. Usually this error occurs when the user described several alternative pneumatic systems, only one of which must be active. The not active PS must be disabled, see Sect. 31.3.2.1 *List of pneumatic systems*.

PS Model: False static values of force and height, air spring "Name".
Static force value is out of the table data interval for the given AS height, Sect. 31.3.1 *Parameters of tabular air springs*. Change height and/or force value (F_0, h_0), Figure 31.34.

PS Model: Force table not correct
The force table must include data for at least two pressure and two height values.
The height and pressure must increase with index of row and column.
See Sect. 31.2.3.1.2 *Tabular data format*.

PS Model: Volume table not correct
The volume table must include data for at least one pressure and two height values.
The volume value must increase with the growth of both the pressure and the height.
The height and pressure must increase with index of row and column.
See Sect. 31.2.3.1.2 *Tabular data format*.

31.5.2. Errors during simulation process

PS Sim: Iterations do not converge
Solving nonlinear pneumatic equations fails. Try to change the pipeline model.

PS Sim: Negative pressure. Interruption
Negative pressure in one of the nodes detected. Simulation cannot be continued. Try to change the pipeline model. If the error appears anyway, please contact UM developers.

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